UNIT 3

Quadrilaterals and Circles

Two-dimensional shapes such as quadrilaterals and circles can be used to describe and model the world around us. In this unit, you will learn about the properties of quadrilaterals and circles and how these two-dimensional figures can be transformed.

Chapter 8
Quadrilaterals

Chapter 9
Transformations

Chapter 10
Circles

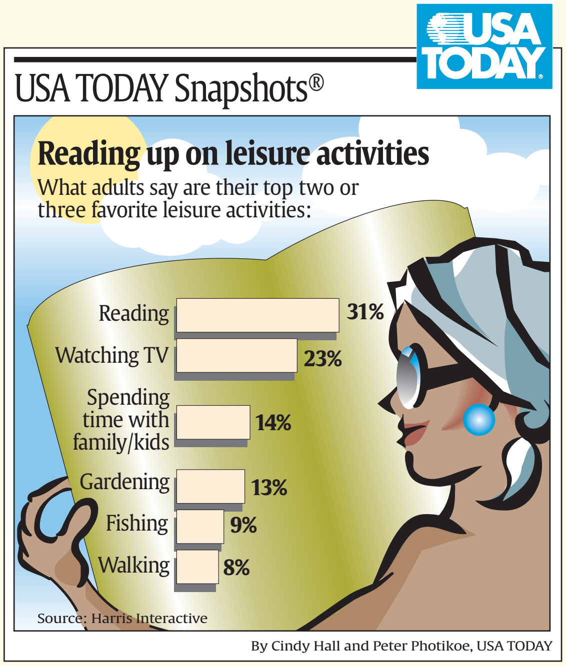
WebQuest Internet Project

"Geocaching" Sends Folks on a Scavenger Hunt

Source: *USA TODAY,* July 26, 2001

"N42 DEGREES 02.054 W88 DEGREES 12.329 – Forget the poison ivy and needle-sharp brambles.

Dave April is a man on a mission. Clutching a palm-size Global Positioning System (GPS) receiver in one hand and a computer printout with latitude and longitude coordinates in the other, the 37-year-old software developer trudges doggedly through a suburban Chicago forest preserve, intent on finding a geek's version of buried treasure." Geocaching is one of the many new ways that people are spending their leisure time. In this project, you will use quadrilaterals, circles, and geometric transformations to give clues for a treasure hunt.



Log on to www.geometryonline.com/webquest. Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 3.

Lesson	8-6	9-1	10-1
Page	444	469	527

Chapter 8

Quadrilaterals

What You'll Learn

- **Lesson 8-1** Investigate interior and exterior angles of polygons.
- **Lessons 8-2 and 8-3** Recognize and apply the properties of parallelograms.
- **Lessons 8-4 through 8-6** Recognize and apply the properties of rectangles, rhombi, squares, and trapezoids.
- **Lesson 8-7** Position quadrilaterals for use in coordinate proof.

Key Vocabulary

- **parallelogram** (p. 411)
- **rectangle** (p. 424)
- **rhombus** (p. 431)
- **square** (p. 432)
- **trapezoid** (p. 439)

Why It's Important

Several different geometric shapes are examples of quadrilaterals. These shapes each have individual characteristics. A rectangle is a type of quadrilateral. Tennis courts are rectangles, and the properties of the rectangular court are used in the game. *You will learn more about tennis courts in Lesson 8-4.*

Michael Newman/PhotoEdit

Getting Started

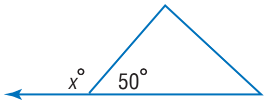
Prerequisite Skills

To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 8.

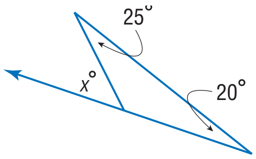
For Lesson 8-1 — Exterior Angles of Triangles

Find x for each figure. *(For review, see Lesson 4-2.)*

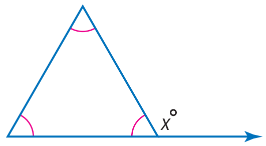
1.



2.



3.



For Lessons 8-4 and 8-5 — Perpendicular Lines

Find the slopes of $\overline{RS}$ and $\overline{TS}$ for the given points, R, T, and S. Determine whether $\overline{RS}$ and $\overline{TS}$ are *perpendicular* or *not perpendicular*. *(For review, see Lesson 3-6.)*

4. $R(4, 3), S(-1, 10), T(13, 20)$

5. $R(-9, 6), S(3, 8), T(1, 20)$

6. $R(-6, -1), S(5, 3), T(2, 5)$

7. $R(-6, 4), S(-3, 8), T(5, 2)$

For Lesson 8-7 — Slope

Write an expression for the slope of a segment given the coordinates of the endpoints. *(For review, see Lesson 3-3.)*

8. $\left(\frac{c}{2}, \frac{d}{2}\right), (-c, d)$

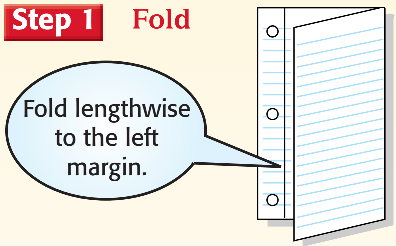
9. $(0, a), (b, 0)$

10. $(-a, c), (-c, a)$

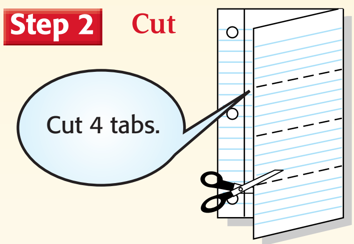
FOLDABLES™ Study Organizer

Quadrilaterals Make this Foldable to help you organize information about quadrilaterals. Begin with a sheet of notebook paper.

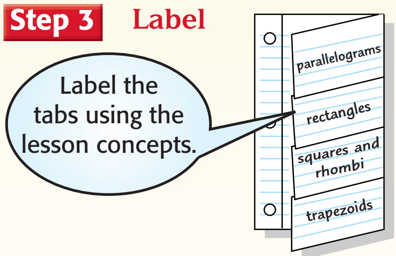
Step 1 Fold



Step 2 Cut



Step 3 Label

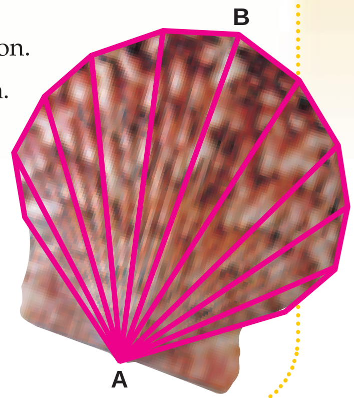


Reading and Writing As you read and study the chapter, use your Foldable to take notes, define terms, and record concepts about quadrilaterals.

8-1 Angles of Polygons

What You'll Learn

- Find the sum of the measures of the interior angles of a polygon.
- Find the sum of the measures of the exterior angles of a polygon.



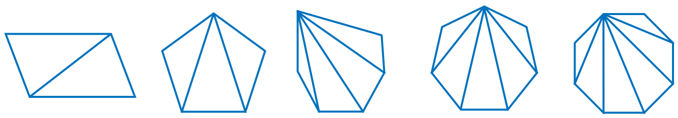
How does a scallop shell illustrate the angles of polygons?

This scallop shell resembles a 12-sided polygon with diagonals drawn from one of the vertices. A **diagonal** of a polygon is a segment that connects any two nonconsecutive vertices. For example, $\overline{AB}$ is one of the diagonals of this polygon.

Vocabulary

- diagonal

SUM OF MEASURES OF INTERIOR ANGLES Polygons with more than three sides have diagonals. The polygons below show all of the possible diagonals drawn from one vertex.



quadrilateral pentagon hexagon heptagon octagon

Study Tip

Look Back
To review the **sum of the measures of the angles of a triangle**, see Lesson 4-2.

In each case, the polygon is separated into triangles. Each angle of the polygon is made up of one or more angles of triangles. The sum of the measures of the angles of each polygon can be found by adding the measures of the angles of the triangles. Since the sum of the measures of the angles in a triangle is 180, we can easily find this sum. Make a table to find the sum of the angle measures for several convex polygons.

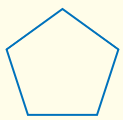
Convex Polygon	Number of Sides	Number of Triangle	Sum of Angle Measures
triangle	3	1	(1 · 180) or 180
quadrilateral	4	2	(2 · 180) or 360
pentagon	5	3	(3 · 180) or 540
hexagon	6	4	(4 · 180) or 720
heptagon	7	5	(5 · 180) or 900
octagon	8	6	(6 · 180) or 1080

Look for a pattern in the sum of the angle measures. In each case, the sum of the angle measures is 2 less than the number of sides in the polygon times 180. So in an n-gon, the sum of the angle measures will be $(n - 2)180$ or $180(n - 2)$.

Theorem 8.1

Interior Angle Sum Theorem If a convex polygon has n sides and S is the sum of the measures of its interior angles, then $S = 180(n - 2)$.

Example:

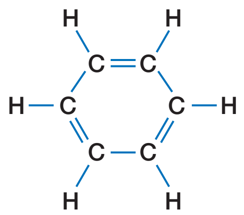


$n = 5$
$S = 180(n - 2)$
$= 180(5 - 2)$ or 540

Glencoe photo

Example 1 Interior Angles of Regular Polygons

CHEMISTRY **The benzene molecule, C_6H_6, consists of six carbon atoms in a regular hexagonal pattern with a hydrogen atom attached to each carbon atom. Find the sum of the measures of the interior angles of the hexagon.**



Since the molecule is a convex polygon, we can use the Interior Angle Sum Theorem.

$S = 180(n - 2)$ — Interior Angle Sum Theorem

$= 180(6 - 2)$ — $n = 6$

$= 180(4)$ or 720 — Simplify.

The sum of the measures of the interior angles is 720.

The Interior Angle Sum Theorem can also be used to find the number of sides in a regular polygon if you are given the measure of one interior angle.

Example 2 Sides of a Polygon

The measure of an interior angle of a regular polygon is 108. Find the number of sides in the polygon.

Use the Interior Angle Sum Theorem to write an equation to solve for n, the number of sides.

$S = 180(n - 2)$ — Interior Angle Sum Theorem

$(108)n = 180(n - 2)$ — $S = 108n$

$108n = 180n - 360$ — Distributive Property

$0 = 72n - 360$ — Subtract $108n$ from each side.

$360 = 72n$ — Add 360 to each side.

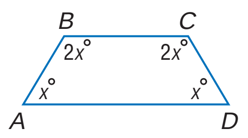
$5 = n$ — Divide each side by 72.

The polygon has 5 sides.

In Example 2, the Interior Angle Sum Theorem was applied to a regular polygon. In Example 3, we will apply this theorem to a quadrilateral that is not a regular polygon.

Example 3 Interior Angles

ALGEBRA **Find the measure of each interior angle.**



Since $n = 4$, the sum of the measures of the interior angles is $180(4 - 2)$ or 360. Write an equation to express the sum of the measures of the interior angles of the polygon.

$360 = m\angle A + m\angle B + m\angle C + m\angle D$ — Sum of measures of angles

$360 = x + 2x + 2x + x$ — Substitution

$360 = 6x$ — Combine like terms.

$60 = x$ — Divide each side by 6.

Use the value of x to find the measure of each angle.

$m\angle A = 60$, $m\angle B = 2 \cdot 60$ or 120, $m\angle C = 2 \cdot 60$ or 120, and $m\angle D = 60$.

SUM OF MEASURES OF EXTERIOR ANGLES

The Interior Angle Sum Theorem relates the interior angles of a convex polygon to the number of sides. Is there a relationship among the exterior angles of a convex polygon?

Study Tip

Look Back

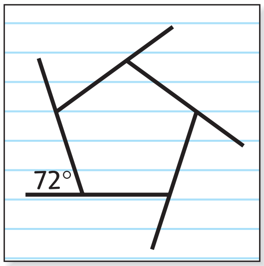
To review **exterior angles**, see Lesson 4-2.

Geometry Activity

Sum of the Exterior Angles of a Polygon

Collect Data

- Draw a triangle, a convex quadrilateral, a convex pentagon, a convex hexagon, and a convex heptagon.
- Extend the sides of each polygon to form exactly one exterior angle at each vertex.
- Use a protractor to measure each exterior angle of each polygon and record it on your drawing.



Analyze the Data

1. Copy and complete the table.

Polygon	triangle	quadrilateral	pentagon	hexagon	heptagon
number of exterior angles					
sum of measure of exterior angles					

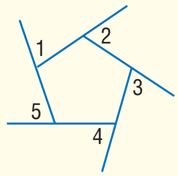
2. What conjecture can you make?

The Geometry Activity suggests Theorem 8.2.

Theorem 8.2

Exterior Angle Sum Theorem If a polygon is convex, then the sum of the measures of the exterior angles, one at each vertex, is 360.

Example:

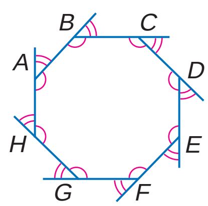


$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360$

You will prove Theorem 8.2 in Exercise 42.

Example 4 *Exterior Angles*

Find the measures of an exterior angle and an interior angle of convex regular octagon *ABCDEFGH*.



At each vertex, extend a side to form one exterior angle. The sum of the measures of the exterior angles is 360. A convex regular octagon has 8 congruent exterior angles.

$8n = 360$ — n = measure of each exterior angle

$n = 45$ — Divide each side by 8.

The measure of each exterior angle is 45. Since each exterior angle and its corresponding interior angle form a linear pair, the measure of the interior angle is $180 - 45$ or 135.

Check for Understanding

Concept Check

1. **Explain** why the Interior Angle Sum Theorem and the Exterior Angle Sum Theorem only apply to convex polygons.
2. **Determine** whether the Interior Angle Sum Theorem and the Exterior Angle Sum Theorem apply to polygons that are not regular. Explain.
3. **OPEN ENDED** Draw a regular convex polygon and a convex polygon that is not regular with the same number of sides. Find the sum of the interior angles for each.

Guided Practice

Find the sum of the measures of the interior angles of each convex polygon.

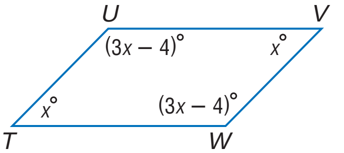
4. pentagon
5. dodecagon

The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.

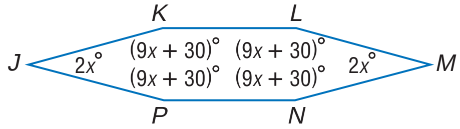
6. 60
7. 90

ALGEBRA **Find the measure of each interior angle.**

8.



9.

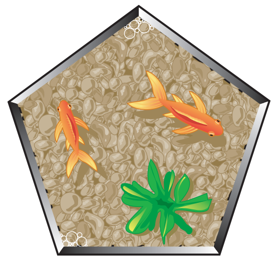


Find the measures of an exterior angle and an interior angle given the number of sides of each regular polygon.

10. 6
11. 18

Application

12. **AQUARIUMS** The regular polygon at the right is the base of a fish tank. Find the sum of the measures of the interior angles of the pentagon.



Practice and Apply

Homework Help

For Exercises	See Examples
13–20	1
21–26	2
27–34	3
35–44	4

Extra Practice
See page 769.

Find the sum of the measures of the interior angles of each convex polygon.

13. 32-gon
14. 18-gon
15. 19-gon
16. 27-gon
17. $4y$-gon
18. $2x$-gon

19. **GARDENING** Carlotta is designing a garden for her backyard. She wants a flower bed shaped like a regular octagon. Find the sum of the measures of the interior angles of the octagon.

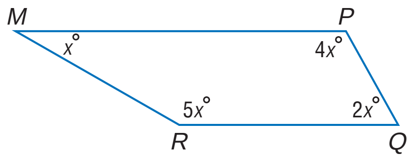
20. **GAZEBOS** A company is building regular hexagonal gazebos. Find the sum of the measures of the interior angles of the hexagon.

The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.

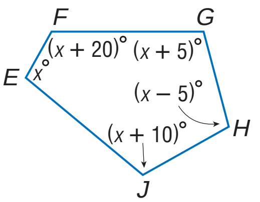
21. 140
22. 170
23. 160
24. 165
25. $157\frac{1}{2}$
26. $176\frac{2}{5}$

ALGEBRA **Find the measure of each interior angle using the given information.**

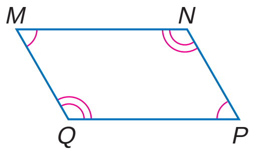
27.



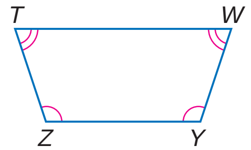
28.



29. parallelogram $MNPQ$ with $m\angle M = 10x$ and $m\angle N = 20x$



30. isosceles trapezoid $TWYZ$ with $\angle Z \cong \angle Y$, $m\angle Z = 30x$, $\angle T \cong \angle W$, and $m\angle T = 20x$



31. decagon in which the measures of the interior angles are $x + 5$, $x + 10$, $x + 20$, $x + 30$, $x + 35$, $x + 40$, $x + 60$, $x + 70$, $x + 80$, and $x + 90$

32. polygon $ABCDE$ with $m\angle A = 6x$, $m\angle B = 4x + 13$, $m\angle C = x + 9$, $m\angle D = 2x - 8$, and $m\angle E = 4x - 1$

33. quadrilateral in which the measure of each consecutive angle is a consecutive multiple of x

34. quadrilateral in which the measure of each consecutive angle increases by 10°

Find the measures of each exterior angle and each interior angle for each regular polygon.

35. decagon

36. hexagon

37. nonagon

38. octagon

Find the measures of an interior angle and an exterior angle given the number of sides of each regular polygon. Round to the nearest tenth if necessary.

39. 11

40. 7

41. 12

42. **PROOF** Use algebra to prove the Exterior Angle Sum Theorem.

43. **ARCHITECTURE** The Pentagon building in Washington, D.C., was designed to resemble a regular pentagon. Find the measure of an interior angle and an exterior angle of the courtyard.

More About. . .

Architecture

Thomas Jefferson's home, Monticello, features a dome on an octagonal base. The architectural elements on either side of the dome were based on a regular octagon.

Source: www.monticello.org

44. **ARCHITECTURE** Compare the dome to the architectural elements on each side of the dome. Are the interior and exterior angles the same? Find the measures of the interior and exterior angles.

45. **CRITICAL THINKING** Two formulas can be used to find the measure of an interior angle of a regular polygon: $s = \frac{180(n-2)}{n}$ and $s = 180 - \frac{360}{n}$. Show that these are equivalent.

46. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

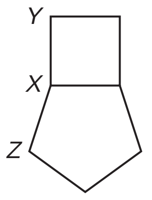
How does a scallop shell illustrate the angles of polygons?

Include the following in your answer:

- explain how triangles are related to the Interior Angle Sum Theorem, and
- describe how to find the measure of an exterior angle of a polygon.

Standardized Test Practice

47. A regular pentagon and a square share a mutual vertex X. The sides $\overline{XY}$ and $\overline{XZ}$ are sides of a third regular polygon with a vertex at X. How many sides does this polygon have?



Ⓐ 19 Ⓑ 20

Ⓒ 28 Ⓓ 32

48. **GRID IN** If $6x + 3y = 48$ and $\frac{9y}{2x} = 9$, then $x = $?

Maintain Your Skills

Mixed Review

In $\triangle ABC$, given the lengths of the sides, find the measure of the given angle to the nearest tenth. *(Lesson 7-7)*

49. $a = 6, b = 9, c = 11; m\angle C$

50. $a = 15.5, b = 23.6, c = 25.1; m\angle B$

51. $a = 47, b = 53, c = 56; m\angle A$

52. $a = 12, b = 14, c = 16; m\angle C$

Solve each $\triangle FGH$ described below. Round angle measures to the nearest degree and side measures to the nearest tenth. *(Lesson 7-6)*

53. $f = 15, g = 17, m\angle F = 54$

54. $m\angle F = 47, m\angle H = 78, g = 31$

55. $m\angle G = 56, m\angle H = 67, g = 63$

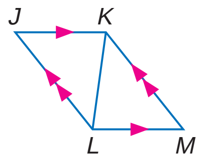
56. $g = 30.7, h = 32.4, m\angle G = 65$

57. **PROOF** Write a two-column proof. *(Lesson 4-5)*

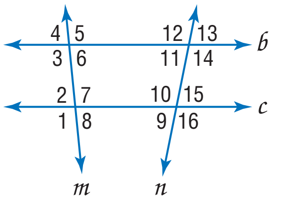
Given: $\overline{JL} \parallel \overline{KM}$

$\overline{JK} \parallel \overline{LM}$

Prove: $\triangle JKL \cong \triangle MLK$



State the transversal that forms each pair of angles. Then identify the special name for the angle pair. *(Lesson 3-1)*



58. $\angle 3$ and $\angle 11$

59. $\angle 6$ and $\angle 7$

60. $\angle 8$ and $\angle 10$

61. $\angle 12$ and $\angle 16$

Getting Ready for the Next Lesson

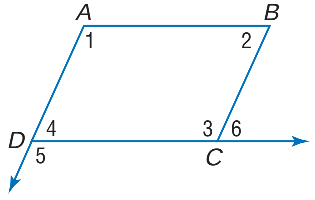
PREREQUISITE SKILL **In the figure, $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$. Name all pairs of angles for each type indicated.** *(To review **angles formed by parallel lines and a transversal**, see Lesson 3-1.)*

62. consecutive interior angles

63. alternate interior angles

64. corresponding angles

65. alternate exterior angles



Spreadsheet Investigation

A Follow-Up of Lesson 8-1

Angles of Polygons

It is possible to find the interior and exterior measurements along with the sum of the interior angles of any regular polygon with n number of sides using a spreadsheet.

Example

Design a spreadsheet using the following steps.

- Label the columns as shown in the spreadsheet below.
- Enter the digits 3–10 in the first column.
- The number of triangles formed by diagonals from the same vertex in a polygon is 2 less than the number of sides. Write a formula for Cell B1 to subtract 2 from each number in Cell A1.
- Enter a formula for Cell C1 so the spreadsheet will find the sum of the measures of the interior angles. Remember that the formula is $S = (n - 2)180$.
- Continue to enter formulas so that the indicated computation is performed. Then, copy each formula through Row 9. The final spreadsheet will appear as below.

Polygons and Angles

	A	B	C	D	E	F	G
1	Number of Sides	Number of Triangles	Sum of Measures of Interior Angles	Measure of Each Interior Angle	Measure of Each Exterior Angle	Sum of Measures of Exterior Angles	
2	3	1	180	60	120	360	
3	4	2	360	90	90	360	
4	5	3	540	108	72	360	
5	6	4	720	120	60	360	
6	7	5	900	128.57	51.43	360	
7	8	6	1080	135	45	360	
8	9	7	1260	140	40	360	
9	10	8	1440	144	36	360	
10							

Sheet1 / Sheet2 / Sheet3

Exercises

1. Write the formula to find the measure of each interior angle in the polygon.
2. Write the formula to find the sum of the measures of the exterior angles.
3. What is the measure of each interior angle if the number of sides is 1? 2?
4. Is it possible to have values of 1 and 2 for the number of sides? Explain.

For Exercises 5–8, use the spreadsheet.

5. How many triangles are in a polygon with 15 sides?
6. Find the measure of the exterior angle of a polygon with 15 sides.
7. Find the measure of the interior angle of a polygon with 110 sides.
8. If the measure of the exterior angles is 0, find the measure of the interior angles. Is this possible? Explain.

8-2 Parallelograms

What You'll Learn

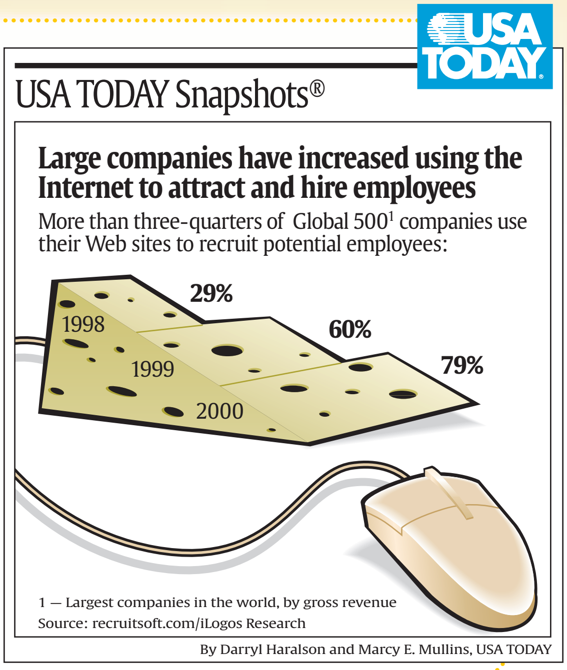
- Recognize and apply properties of the sides and angles of parallelograms.
- Recognize and apply properties of the diagonals of parallelograms.

Vocabulary

- parallelogram

How are parallelograms used to represent data?

The graphic shows the percent of Global 500 companies that use the Internet to find potential employees. The top surfaces of the wedges of cheese are all polygons with a similar shape. However, the size of the polygon changes to reflect the data. What polygon is this?



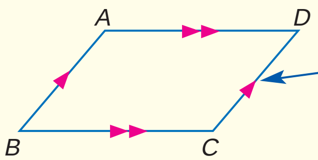
SIDES AND ANGLES OF PARALLELOGRAMS A quadrilateral with parallel opposite sides is called a **parallelogram**.

Study Tip

Reading Math
Recall that the matching arrow marks on the segments mean that the sides are parallel.

Key Concept — Parallelogram

- **Words** A parallelogram is a quadrilateral with both pairs of opposite sides parallel.
- **Symbols** ▱*ABCD*
- **Example**



There are two pairs of parallel sides. $\overline{AB}$ and $\overline{DC}$, $\overline{AD}$ and $\overline{BC}$

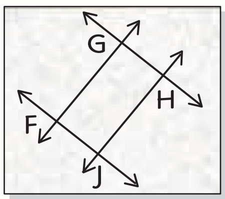
This activity will help you make conjectures about the sides and angles of a parallelogram.

Geometry Activity

Properties of Parallelograms

Make a model

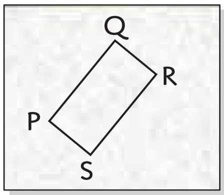
Step 1 Construct two sets of intersecting parallel lines on patty paper. Label the vertices *FGHJ*.



(continued on the next page)

Step 2 Trace *FGHJ*. Label the second parallelogram *PQRS* so ∠*F* and ∠*P* are congruent.

Step 3 Rotate ▱*PQRS* on ▱*FGHJ* to compare sides and angles.



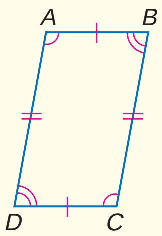
Analyze

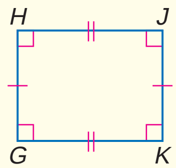
1. List all of the segments that are congruent.
2. List all of the angles that are congruent.
3. Describe the angle relationships you observed.

The Geometry Activity leads to four properties of parallelograms.

Key Concept — *Properties of Parallelograms*

Theorem	Example
8.3 Opposite sides of a parallelogram are congruent. **Abbreviation:** *Opp. sides of ▱ are ≅.*	$\overline{AB} \cong \overline{DC}$ $\overline{AD} \cong \overline{BC}$
8.4 Opposite angles in a parallelogram are congruent. **Abbreviation:** *Opp. ∡ of ▱ are ≅ .*	$\angle A \cong \angle C$ $\angle B \cong \angle D$
8.5 Consecutive angles in a parallelogram are supplementary. **Abbreviation:** *Cons. ∡ in ▱ are suppl.*	$m\angle A + m\angle B = 180$ $m\angle B + m\angle C = 180$ $m\angle C + m\angle D = 180$ $m\angle D + m\angle A = 180$
8.6 If a parallelogram has one right angle, it has four right angles. **Abbreviation:** *If ▱ has 1 rt. ∠, it has 4 rt. ∡.*	$m\angle G = 90$ $m\angle H = 90$ $m\angle J = 90$ $m\angle K = 90$





You will prove Theorems 8.3, 8.5, and 8.6 in Exercises 41, 42, and 43, respectively.

Study Tip

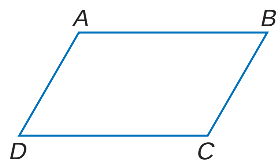
Including a Figure
Theorems are presented in general terms. In a proof, you must include a drawing so that you can refer to segments and angles specifically.

Example 1 *Proof of Theorem 8.4*

Write a two-column proof of Theorem 8.4.

Given: ▱*ABCD*

Prove: $\angle A \cong \angle C$
$\angle D \cong \angle B$

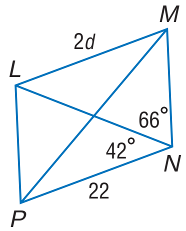


Proof:

Statements	**Reasons**
1. ▱*ABCD*	1. Given
2. $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$	2. Definition of parallelogram
3. ∠*A* and ∠*D* are supplementary. ∠*D* and ∠*C* are supplementary. ∠*C* and ∠*B* are supplementary.	3. If parallel lines are cut by a transversal, consecutive interior angles are supplementary.
4. $\angle A \cong \angle C$ $\angle D \cong \angle B$	4. Supplements of the same angles are congruent.

Example 2 Properties of Parallelograms

ALGEBRA **Quadrilateral *LMNP* is a parallelogram. Find $m\angle PLM$, $m\angle LMN$, and *d*.**



$m\angle MNP = 66 + 42$ or 108 — Angle Addition Theorem

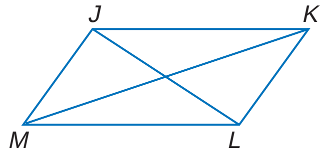
$\angle PLM \cong \angle MNP$ — Opp. ∠s of ▱ are ≅.
$m\angle PLM = m\angle MNP$ — Definition of congruent angles
$m\angle PLM = 108$ — Substitution

$m\angle PLM + m\angle LMN = 180$ — Cons. ∠s of ▱ are suppl.
$108 + m\angle LMN = 180$ — Substitution
$m\angle LMN = 72$ — Subtract 108 from each side.

$\overline{LM} \cong \overline{PN}$ — Opp. sides of ▱ are ≅.
$LM = PN$ — Definition of congruent segments
$2d = 22$ — Substitution
$d = 11$ — Substitution

DIAGONALS OF PARALLELOGRAMS

In parallelogram *JKLM*, $\overline{JL}$ and $\overline{KM}$ are diagonals. Theorem 8.7 states the relationship between diagonals of a parallelogram.

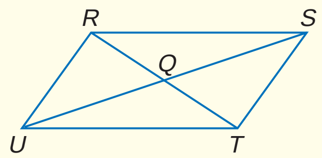


Theorem 8.7

The diagonals of a parallelogram bisect each other.

Abbreviation: *Diag. of ▱ bisect each other.*

Example: $\overline{RQ} \cong \overline{QT}$ and $\overline{SQ} \cong \overline{QU}$



You will prove Theorem 8.7 in Exercise 44.

Standardized Test Practice

Example 3 Diagonals of a Parallelogram

Multiple-Choice Test Item

What are the coordinates of the intersection of the diagonals of parallelogram *ABCD* with vertices *A*(2, 5), *B*(6, 6), *C*(4, 0), and *D*(0, –1)?

(A) (4, 2) (B) (4.5, 2) (C) $\left(\frac{7}{6}, \frac{-5}{2}\right)$ (D) (3, 2.5)

Read the Test Item

Since the diagonals of a parallelogram bisect each other, the intersection point is the midpoint of $\overline{AC}$ and $\overline{BD}$.

Solve the Test Item

Find the midpoint of $\overline{AC}$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + 4}{2}, \frac{5 + 0}{2}\right) \quad \text{Midpoint Formula}$$

$$= (3, 2.5)$$

The coordinates of the intersection of the diagonals of parallelogram *ABCD* are (3, 2.5). The answer is D.

The Princeton Review

Test-Taking Tip

Check Answers Always check your answer. To check the answer to this problem, find the coordinates of the midpoint of $\overline{BD}$.

www.geometryonline.com/extra_examples

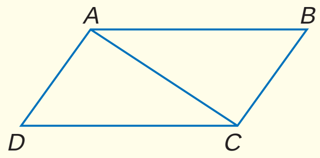
Theorem 8.8 describes another characteristic of the diagonals of a parallelogram.

Theorem 8.8

Each diagonal of a parallelogram separates the parallelogram into two congruent triangles.

Abbreviation: *Diag. separates ▱ into 2 ≅ △s.*

Example: $\triangle ACD \cong \triangle CAB$



You will prove Theorem 8.8 in Exercise 45.

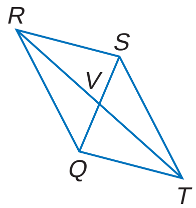
Check for Understanding

Concept Check

1. **Describe** the characteristics of the sides and angles of a parallelogram.
2. **Describe** the properties of the diagonals of a parallelogram.
3. **OPEN ENDED** Draw a parallelogram with one side twice as long as another side.

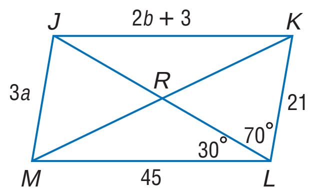
Guided Practice

Complete each statement about ▱*QRST*. Justify your answer.



4. $\overline{SV} \cong$ __?__
5. $\triangle VRS \cong$ __?__
6. $\angle TSR$ is supplementary to __?__.

Use ▱*JKLM* to find each measure or value if $JK = 2b + 3$ and $JM = 3a$.

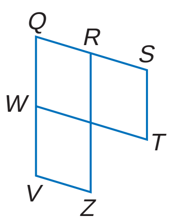


7. $m\angle MJK$
8. $m\angle JML$
9. $m\angle JKL$
10. $m\angle KJL$
11. a
12. b

PROOF **Write the indicated type of proof.**

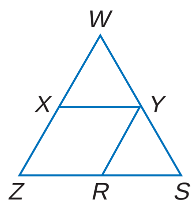
13. two-column

Given: ▱*VZRQ* and ▱*WQST*
Prove: $\angle Z \cong \angle T$



14. paragraph

Given: ▱*XYRZ*, $\overline{WZ} \cong \overline{WS}$
Prove: $\angle XYR \cong \angle S$



Standardized Test Practice

15. **MULTIPLE CHOICE** Find the coordinates of the intersection of the diagonals of parallelogram *GHJK* with vertices $G(-3, 4)$, $H(1, 1)$, $J(3, -5)$, and $K(-1, -2)$.

Ⓐ (0, 0.5)　Ⓑ (6, −1)　Ⓒ (0, −0.5)　Ⓓ (5, 0)

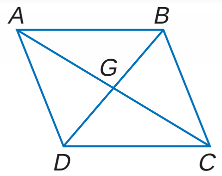
Practice and Apply

Homework Help

For Exercises	See Examples
16–33	2
34–40	3
41–47	1

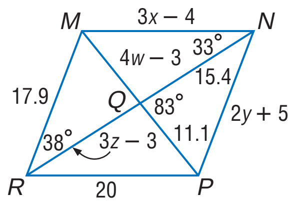
Extra Practice
See page 769.

Complete each statement about ▱*ABCD*. Justify your answer.



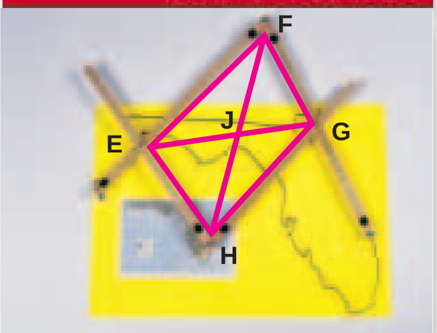
16. $\angle DAB \cong$ __?__
17. $\angle ABD \cong$ __?__
18. $\overline{AB} \parallel$ __?__
19. $\overline{BG} \cong$ __?__
20. $\triangle ABD \cong$ __?__
21. $\angle ACD \cong$ __?__

ALGEBRA **Use ▱*MNPR* to find each measure or value.**



22. $m\angle MNP$
23. $m\angle NRP$
24. $m\angle RNP$
25. $m\angle RMN$
26. $m\angle MQN$
27. $m\angle MQR$
28. x
29. y
30. w
31. z

More About. . .



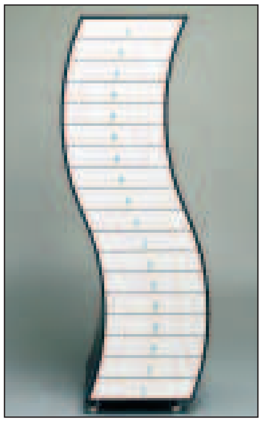
Drawing

The pantograph was used as a primitive copy machine. The device makes an exact replica as the user traces over a figure.

Source: www.infoplease.com

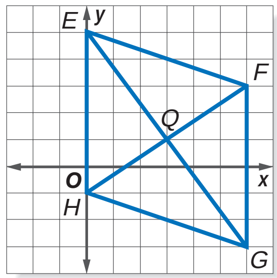
DRAWING **For Exercises 32 and 33, use the following information.**
The frame of a pantograph is a parallelogram.

32. Find x and EG if $EJ = 2x + 1$ and $JG = 3x$.
33. Find y and FH if $HJ = \frac{1}{2}y + 2$ and $JF = y - \frac{1}{2}$.
34. **DESIGN** The chest of drawers shown at the right is called *Side 2*. It was designed by Shiro Kuramata. Describe the properties of parallelograms the artist used to place each drawer pull.

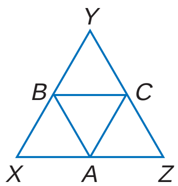


35. **ALGEBRA** Parallelogram $ABCD$ has diagonals $\overline{AC}$ and $\overline{DB}$ that intersect at point P. If $AB = 3a + 18$, $AC = 12a$, $PB = a + 2b$, and $PD = 3b + 1$, find a, b, and DB.
36. **ALGEBRA** In parallelogram $ABCD$, $AB = 2x + 5$, $m\angle BAC = 2y$, $m\angle B = 120$, $m\angle CAD = 21$, and $CD = 21$. Find x and y.

COORDINATE GEOMETRY **For Exercises 37–39, refer to ▱*EFGH*.**



37. Use the Distance Formula to verify that the diagonals bisect each other.
38. Determine whether the diagonals of this parallelogram are congruent.
39. Find the slopes of $\overline{EH}$ and $\overline{EF}$. Are the consecutive sides perpendicular? Explain.
40. Determine the relationship among $ACBX$, $ABYC$, and $ABCZ$ if $\triangle XYZ$ is equilateral and A, B, and C are midpoints of $\overline{XZ}$, $\overline{XY}$, and $\overline{ZY}$, respectively.

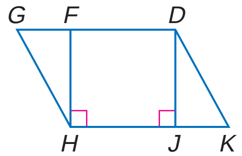


PROOF **Write the indicated type of proof.**

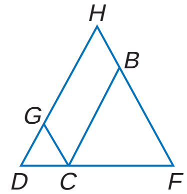
41. two-column proof of Theorem 8.3
42. two-column proof of Theorem 8.5
43. paragraph proof of Theorem 8.6
44. paragraph proof of Theorem 8.7
45. two-column proof of Theorem 8.8

PROOF **Write a two-column proof.**

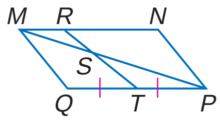
46. Given: $\square DGHK$, $\overline{FH} \perp \overline{GD}$, $\overline{DJ} \perp \overline{HK}$
Prove: $\triangle DJK \cong \triangle HFG$



47. Given: $\square BCGH$, $\overline{HD} \cong \overline{FD}$
Prove: $\angle F \cong \angle GCB$



48. CRITICAL THINKING Find the ratio of MS to SP, given that $MNPQ$ is a parallelogram with $MR = \frac{1}{4}MN$.



49. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How are parallelograms used to represent data?

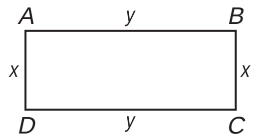
Include the following in your answer:

- properties of parallelograms, and
- a display of the data in the graphic with a different parallelogram.

Standardized Test Practice

50. SHORT RESPONSE Two consecutive angles of a parallelogram measure $(3x + 42)°$ and $(9x - 18)°$. Find the measures of the angles.

51. ALGEBRA The perimeter of the rectangle $ABCD$ is equal to p and $x = \frac{y}{5}$. What is the value of y in terms of p?



(A) $\frac{p}{3}$ (B) $\frac{5p}{12}$ (C) $\frac{5p}{8}$ (D) $\frac{5p}{6}$

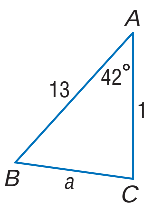
Maintain Your Skills

Find the sum of the measures of the interior angles of each convex polygon. *(Lesson 8-1)*

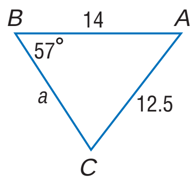
52. 14-gon **53.** 22-gon **54.** 17-gon **55.** 36-gon

Determine whether the *Law of Sines* or the *Law of Cosines* should be used to solve each triangle. Then solve each triangle. Round to the nearest tenth. *(Lesson 7-7)*

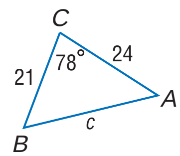
56.



57.



58.



Use Pascal's Triangle for Exercises 59 and 60. *(Lesson 6-6)*

59. Find the sum of the first 30 numbers in the outside diagonal of Pascal's triangle.

60. Find the sum of the first 70 numbers in the second diagonal.

Getting Ready for the Next Lesson

PREREQUISITE SKILL The vertices of a quadrilateral are $A(-5, -2)$, $B(-2, 5)$, $C(2, -2)$, and $D(-1, -9)$. Determine whether each segment is a side or a diagonal of the quadrilateral, and find the slope of each segment. *(To review **slope**, see Lesson 3-3.)*

61. $\overline{AB}$ **62.** $\overline{BD}$ **63.** $\overline{CD}$

CONTENTS

8-3 Tests for Parallelograms

What You'll Learn

- Recognize the conditions that ensure a quadrilateral is a parallelogram.
- Prove that a set of points forms a parallelogram in the coordinate plane.

How are parallelograms used in architecture?

The roof of the covered bridge appears to be a parallelogram. Each pair of opposite sides looks like they are the same length. How can we know for sure if this shape is really a parallelogram?

CONDITIONS FOR A PARALLELOGRAM By definition, the opposite sides of a parallelogram are parallel. So, if a quadrilateral has each pair of opposite sides parallel it is a parallelogram. Other tests can be used to determine if a quadrilateral is a parallelogram.

Geometry Activity

Testing for a Parallelogram

Model

- Cut two straws to one length and two other straws to a different length.
- Connect the straws by inserting a pipe cleaner in one end of each size of straw to form a quadrilateral like the one shown at the right.
- Shift the sides to form quadrilaterals of different shapes.

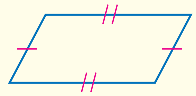
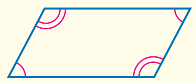
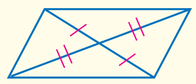
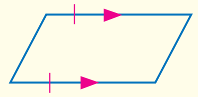
Analyze

1. Measure the distance between the opposite sides of the quadrilateral in at least three places. Repeat this process for several figures. What can you conclude about opposite sides?
2. Classify the quadrilaterals that you formed.
3. Compare the measures of pairs of opposite sides.
4. Measure the four angles in several of the quadrilaterals. What relationships do you find?

Make a Conjecture

5. What conditions are necessary to verify that a quadrilateral is a parallelogram?

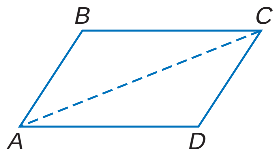
Key Concept — Proving Parallelograms

	Theorem	Example
8.9	If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. **Abbreviation:** *If both pairs of opp. sides are ≅, then quad. is ▱.*	
8.10	If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. **Abbreviation:** *If both pairs of opp. ∠s are ≅, then quad. is ▱.*	
8.11	If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. **Abbreviation:** *If diag. bisect each other, then quad. is ▱.*	
8.12	If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram. **Abbreviation:** *If one pair of opp. sides is ∥ and ≅, then the quad. is a ▱.*	

You will prove Theorems 8.9, 8.11, and 8.12 in Exercises 39, 40, and 41, respectively.

Example 1 Write a Proof

PROOF **Write a paragraph proof for Theorem 8.10**



Given: $\angle A \cong \angle C$, $\angle B \cong \angle D$

Prove: $ABCD$ is a parallelogram.

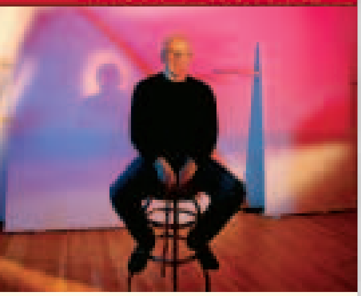
Paragraph Proof:

Because two points determine a line, we can draw $\overline{AC}$. We now have two triangles. We know the sum of the angle measures of a triangle is 180, so the sum of the angle measures of two triangles is 360. Therefore, $m\angle A + m\angle B + m\angle C + m\angle D = 360$.

Since $\angle A \cong \angle C$ and $\angle B \cong \angle D$, $m\angle A = m\angle C$ and $m\angle B = m\angle D$. Substitute to find that $m\angle A + m\angle A + m\angle B + m\angle B = 360$, or $2(m\angle A) + 2(m\angle B) = 360$. Dividing each side of the equation by 2 yields $m\angle A + m\angle B = 180$. This means that consecutive angles are supplementary and $\overline{AD} \parallel \overline{BC}$.

Likewise, $2m\angle A + 2m\angle D = 360$, or $m\angle A + m\angle D = 180$. These consecutive supplementary angles verify that $\overline{AB} \parallel \overline{DC}$. Opposite sides are parallel, so $ABCD$ is a parallelogram.

More About...



Art

Ellsworth Kelly created *Sculpture for a Large Wall* in 1957. The sculpture is made of 104 aluminum panels. The piece is over 65 feet long, 11 feet high, and 2 feet deep.

Source: www.moma.org

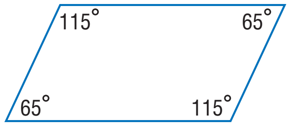
Example 2 Properties of Parallelograms

ART **Some panels in the sculpture appear to be parallelograms. Describe the information needed to determine whether these panels are parallelograms.**

A panel is a parallelogram if both pairs of opposite sides are congruent, or if one pair of opposite sides is congruent and parallel. If the diagonals bisect each other, or if both pairs of opposite angles are congruent, then the panel is a parallelogram.

Example 3 Properties of Parallelograms

Determine whether the quadrilateral is a parallelogram. Justify your answer.



Each pair of opposite angles have the same measure. Therefore, they are congruent. If both pairs of opposite angles are congruent, the quadrilateral is a parallelogram.

A quadrilateral is a parallelogram if any one of the following is true.

Concept Summary — Tests for a Parallelogram

1. Both pairs of opposite sides are parallel. (Definition)
2. Both pairs of opposite sides are congruent. (Theorem 8.9)
3. Both pairs of opposite angles are congruent. (Theorem 8.10)
4. Diagonals bisect each other. (Theorem 8.11)
5. A pair of opposite sides is both parallel and congruent. (Theorem 8.12)

Study Tip

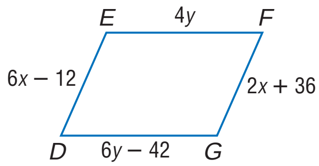
Common Misconceptions

If a quadrilateral meets one of the five tests, it is a parallelogram. All of the properties of parallelograms need not be shown.

Example 4 Find Measures

ALGEBRA **Find x and y so that each quadrilateral is a parallelogram.**

a.

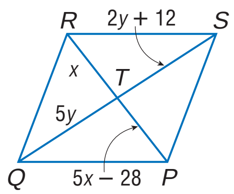


Opposite sides of a parallelogram are congruent.

$\overline{EF} \cong \overline{DG}$	Opp. sides of ▱ are ≅.	$\overline{DE} \cong \overline{FG}$	Opp. sides of ▱ are ≅.
$EF = DG$	Def. of ≅ segments	$DE = FG$	Def. of ≅ segments
$4y = 6y - 42$	Substitution	$6x - 12 = 2x + 36$	Substitution
$-2y = -42$	Subtract $6y$.	$4x = 48$	Subtract $2x$ and add 12.
$y = 21$	Divide by -2.	$x = 12$	Divide by 4.

So, when x is 12 and y is 21, $DEFG$ is a parallelogram.

b.



Diagonals in a parallelogram bisect each other.

$\overline{QT} \cong \overline{TS}$	Opp. sides of ▱ are ≅.	$\overline{RT} \cong \overline{TP}$	Opp. sides of ▱ are ≅.
$QT = TS$	Def. of ≅ segments	$RT = TP$	Def. of ≅ segments
$5y = 2y + 12$	Substitution	$x = 5x - 28$	Substitution
$3y = 12$	Subtract $2y$.	$-4x = -28$	Subtract $5x$.
$y = 4$	Divide by 3.	$x = 7$	Divide by -4.

$PQRS$ is a parallelogram when $x = 7$ and $y = 4$.

PARALLELOGRAMS ON THE COORDINATE PLANE We can use the Distance Formula and the Slope Formula to determine if a quadrilateral is a parallelogram in the coordinate plane.

Study Tip

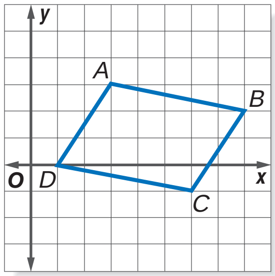
Coordinate Geometry

The Midpoint Formula can also be used to show that a quadrilateral is a parallelogram by Theorem 8.11.

Example 5 Use Slope and Distance

COORDINATE GEOMETRY **Determine whether the figure with the given vertices is a parallelogram. Use the method indicated.**

a. $A(3, 3)$, $B(8, 2)$, $C(6, -1)$, $D(1, 0)$; **Slope Formula**



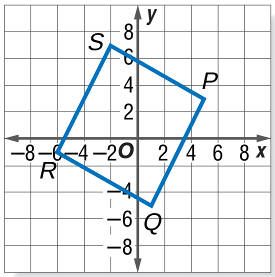
If the opposite sides of a quadrilateral are parallel, then it is a parallelogram.

slope of $\overline{AB} = \frac{2-3}{8-3}$ or $\frac{-1}{5}$ slope of $\overline{DC} = \frac{-1-0}{6-1}$ or $\frac{-1}{5}$

slope of $\overline{AD} = \frac{3-0}{3-1}$ or $\frac{3}{2}$ slope of $\overline{BC} = \frac{-1-2}{6-8}$ or $\frac{3}{2}$

Since opposite sides have the same slope, $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$. Therefore, $ABCD$ is a parallelogram by definition.

b. $P(5, 3)$, $Q(1, -5)$, $R(-6, -1)$, $S(-2, 7)$; **Distance and Slope Formulas**



First use the Distance Formula to determine whether the opposite sides are congruent.

$$PS = \sqrt{[5-(-2)]^2 + (3-7)^2}$$
$$= \sqrt{7^2 + (-4)^2} \text{ or } \sqrt{65}$$

$$QR = \sqrt{[1-(-6)]^2 + [-5-(-1)]^2}$$
$$= \sqrt{7^2 + (-4)^2} \text{ or } \sqrt{65}$$

Since $PS = QR$, $\overline{PS} \cong \overline{QR}$.

Next, use the Slope Formula to determine whether $\overline{PS} \parallel \overline{QR}$.

slope of $\overline{PS} = \frac{3-7}{5-(-2)}$ or $-\frac{4}{7}$ slope of $\overline{QR} = \frac{-5-(-1)}{1-(-6)}$ or $-\frac{4}{7}$

$\overline{PS}$ and $\overline{QR}$ have the same slope, so they are parallel. Since one pair of opposite sides is congruent and parallel, $PQRS$ is a parallelogram.

Check for Understanding

Concept Check

1. **List** and describe four tests for parallelograms.

2. **OPEN ENDED** Draw a parallelogram. Label the congruent angles.

3. **FIND THE ERROR** Carter and Shaniqua are describing ways to show that a quadrilateral is a parallelogram.

Carter	Shaniqua
A quadrilateral is a parallelogram if one pair of opposite sides is congruent and one pair of opposite sides is parallel.	A quadrilateral is a parallelogram if one pair of opposite sides is congruent and parallel.

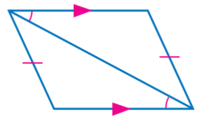
Who is correct? Explain your reasoning.

CONTENTS

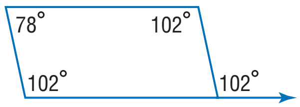
Guided Practice

Determine whether each quadrilateral is a parallelogram. Justify your answer.

4.

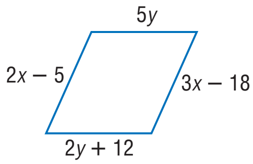


5.

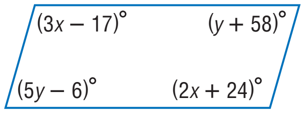


ALGEBRA **Find x and y so that each quadrilateral is a parallelogram.**

6.



7.



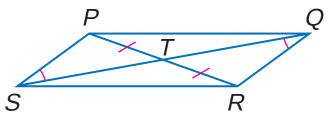
COORDINATE GEOMETRY **Determine whether the figure with the given vertices is a parallelogram. Use the method indicated.**

8. $B(0, 0)$, $C(4, 1)$, $D(6, 5)$, $E(2, 4)$; Slope Formula

9. $A(-4, 0)$, $B(3, 1)$, $C(1, 4)$, $D(-6, 3)$; Distance and Slope Formulas

10. $E(-4, -3)$, $F(4, -1)$, $G(2, 3)$, $H(-6, 2)$; Midpoint Formula

11. **PROOF** Write a two-column proof to prove that $PQRS$ is a parallelogram given that $\overline{PT} \cong \overline{TR}$ and $\angle TSP \cong \angle TQR$.



Application

12. **TANGRAMS** A tangram set consists of seven pieces: a small square, two small congruent right triangles, two large congruent right triangles, a medium-sized right triangle, and a quadrilateral. How can you determine the shape of the quadrilateral? Explain.

Practice and Apply

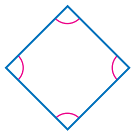
Homework Help

For Exercises	See Examples
13–18	3
19–24	4
25–36	5
37–38	2
39–42	1

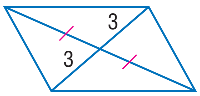
Extra Practice
See page 769.

Determine whether each quadrilateral is a parallelogram. Justify your answer.

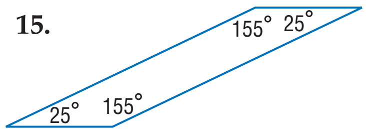
13.



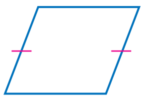
14.



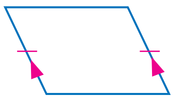
15.



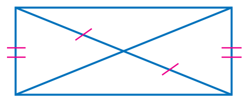
16.



17.

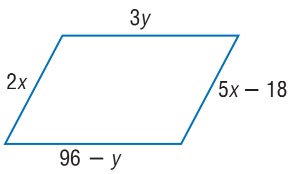


18.

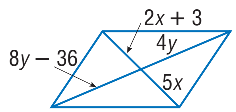


ALGEBRA **Find x and y so that each quadrilateral is a parallelogram.**

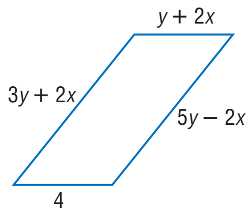
19.



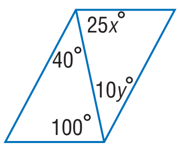
20.



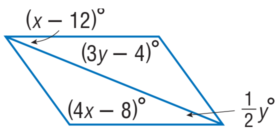
21.



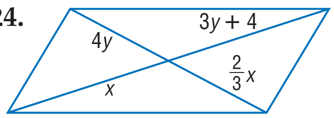
22.



23.



24.



COORDINATE GEOMETRY **Determine whether a figure with the given vertices is a parallelogram. Use the method indicated.**

25. $B(-6, -3)$, $C(2, -3)$, $E(4, 4)$, $G(-4, 4)$; Slope Formula
26. $Q(-3, -6)$, $R(2, 2)$, $S(-1, 6)$, $T(-5, 2)$; Slope Formula
27. $A(-5, -4)$, $B(3, -2)$, $C(4, 4)$, $D(-4, 2)$; Distance Formula
28. $W(-6, -5)$, $X(-1, -4)$, $Y(0, -1)$, $Z(-5, -2)$; Midpoint Formula
29. $G(-2, 8)$, $H(4, 4)$, $J(6, -3)$, $K(-1, -7)$; Distance and Slope Formulas
30. $H(5, 6)$, $J(9, 0)$, $K(8, -5)$, $L(3, -2)$; Distance Formula
31. $S(-1, 9)$, $T(3, 8)$, $V(6, 2)$, $W(2, 3)$; Midpoint Formula
32. $C(-7, 3)$, $D(-3, 2)$, $F(0, -4)$, $G(-4, -3)$; Distance and Slope Formulas

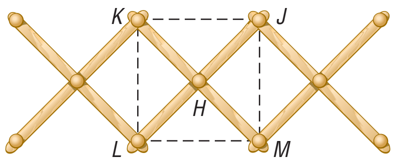
33. Quadrilateral *MNPR* has vertices $M(-6, 6)$, $N(-1, -1)$, $P(-2, -4)$, and $R(-5, -2)$. Determine how to move one vertex to make *MNPR* a parallelogram.

34. Quadrilateral *QSTW* has vertices $Q(-3, 3)$, $S(4, 1)$, $T(-1, -2)$, and $W(-5, -1)$. Determine how to move one vertex to make *QSTW* a parallelogram.

COORDINATE GEOMETRY **The coordinates of three of the vertices of a parallelogram are given. Find the possible coordinates for the fourth vertex.**

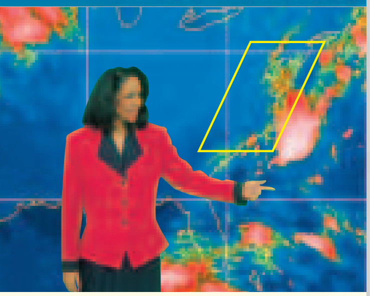
35. $A(1, 4)$, $B(7, 5)$, and $C(4, -1)$.
36. $Q(-2, 2)$, $R(1, 1)$, and $S(-1, -1)$.

37. **STORAGE** Songan purchased an expandable hat rack that has 11 pegs. In the figure, *H* is the midpoint of $\overline{KM}$ and $\overline{JL}$. What type of figure is *JKLM*? Explain.



38. **METEOROLOGY** To show the center of a storm, television stations superimpose a "watchbox" over the weather map. Describe how you know that the watchbox is a parallelogram.

Career Choices



Atmospheric Scientist

Atmospheric scientists, or meteorologists, study weather patterns. They can work for private companies, the Federal Government or television stations.

Online Research For information about a career as an atmospheric scientist, visit: www.geometryonline.com/careers

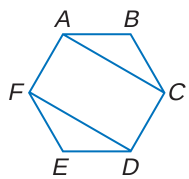
Online Research **Data Update** Each hurricane is assigned a name as the storm develops. What is the name of the most recent hurricane or tropical storm in the Atlantic or Pacific Oceans? Visit www.geometryonline.com/data_update to learn more.

PROOF **Write a two-column proof of each theorem.**

39. Theorem 8.9
40. Theorem 8.11
41. Theorem 8.12

42. Li-Cheng claims she invented a new geometry theorem. *A diagonal of a parallelogram bisects its angles.* Determine whether this theorem is true. Find an example or counterexample.

43. **CRITICAL THINKING** Write a proof to prove that *FDCA* is a parallelogram if *ABCDEF* is a regular hexagon.



44. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are parallelograms used in architecture?

Include the following in your answer:
- the information needed to prove that the roof of the covered bridge is a parallelogram, and
- another example of parallelograms used in architecture.

Standardized Test Practice

45. A parallelogram has vertices at (−2, 2), (1, −6), and (8, 2). Which ordered pair could represent the fourth vertex?

(A) (5, 6) (B) (11, −6) (C) (14, 3) (D) (8, −8)

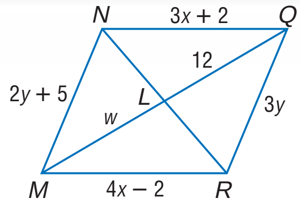
46. ALGEBRA Find the distance between $X(5, 7)$ and $Y(-3, -4)$.

(A) $\sqrt{19}$ (B) $3\sqrt{15}$ (C) $\sqrt{185}$ (D) $5\sqrt{29}$

Maintain Your Skills

Mixed Review

Use ▱*NQRM* to find each measure or value. *(Lesson 8-2)*



47. w

48. x

49. NQ

50. QR

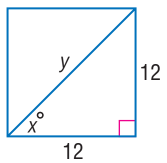
The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon. *(Lesson 8-1)*

51. 135 **52.** 144 **53.** 168

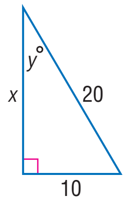
54. 162 **55.** 175 **56.** 175.5

Find x and y. *(Lesson 7-3)*

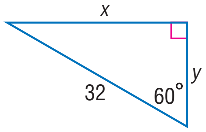
57.



58.



59.



Getting Ready for the Next Lesson

PREREQUISITE SKILL **Use slope to determine whether $\overline{AB}$ and $\overline{BC}$ are *perpendicular* or *not perpendicular*.** *(To review **slope and perpendicularity**, see Lesson 3-3.)*

60. $A(2, 5), B(6, 3), C(8, 7)$

61. $A(-1, 2), B(0, 7), C(4, 1)$

62. $A(0, 4), B(5, 7), C(8, 3)$

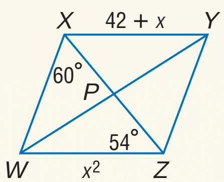
63. $A(-2, -5), B(1, -3), C(-1, 0)$

Practice Quiz 1

Lessons 8-1 through 8-3

1. The measure of an interior angle of a regular polygon is $147\frac{3}{11}$. Find the number of sides in the polygon. *(Lesson 8-1)*

Use ▱*WXYZ* to find each measure. *(Lesson 8-2)*

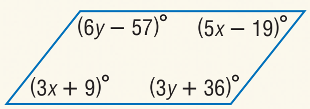


2. WZ = ___?___.

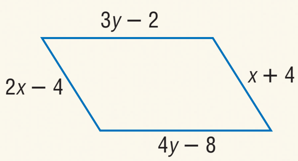
3. $m\angle XYZ$ = ___?___.

ALGEBRA **Find x and y so that each quadrilateral is a parallelogram.** *(Lesson 8-3)*

4.



5.



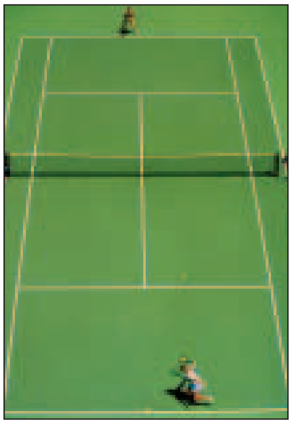
8-4 Rectangles

What You'll Learn

- Recognize and apply properties of rectangles.
- Determine whether parallelograms are rectangles.

How are rectangles used in tennis?

Many sports are played on fields marked by parallel lines. A tennis court has parallel lines at half-court for each player. Parallel lines divide the court for singles and doubles play. The service box is marked by perpendicular lines.



Vocabulary

- rectangle

Study Tip

Rectangles and Parallelograms
A rectangle is a parallelogram, but a parallelogram is not necessarily a rectangle.

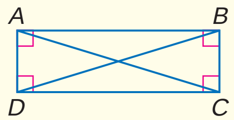
PROPERTIES OF RECTANGLES

A **rectangle** is a quadrilateral with four right angles. Since both pairs of opposite angles are congruent, it follows that it is a special type of parallelogram. Thus, a rectangle has all the properties of a parallelogram. Because the right angles make a rectangle a rigid figure, the diagonals are also congruent.

Theorem 8.13

If a parallelogram is a rectangle, then the diagonals are congruent.

Abbreviation: *If ▱ is rectangle, diag. are ≅.*



$\overline{AC} \cong \overline{BD}$

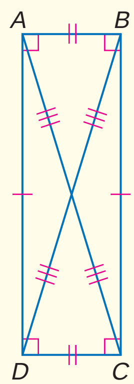
You will prove Theorem 8.13 in Exercise 40.

If a quadrilateral is a rectangle, then the following properties are true.

Key Concept — *Rectangle*

Words A rectangle is a quadrilateral with four right angles.

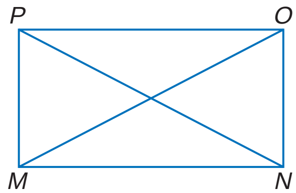
Properties	Examples
1. Opposite sides are congruent and parallel.	$\overline{AB} \cong \overline{DC}$, $\overline{BC} \cong \overline{AD}$; $\overline{AB} \parallel \overline{DC}$, $\overline{BC} \parallel \overline{AD}$
2. Opposite angles are congruent.	$\angle A \cong \angle C$ $\angle B \cong \angle D$
3. Consecutive angles are supplementary.	$m\angle A + m\angle B = 180$ $m\angle B + m\angle C = 180$ $m\angle C + m\angle D = 180$ $m\angle D + m\angle A = 180$
4. Diagonals are congruent and bisect each other.	$\overline{AC}$ and $\overline{BD}$ bisect each other. $\overline{AC} \cong \overline{BD}$
5. All four angles are right angles.	$m\angle DAB = m\angle BCD = m\angle ABC = m\angle ADC = 90$



Simon Bruty/Getty Images

Example 1 Diagonals of a Rectangle

ALGEBRA **Quadrilateral *MNOP* is a rectangle. If $MO = 6x + 14$ and $PN = 9x + 5$, find x.**



The diagonals of a rectangle are congruent, so $\overline{MO} \cong \overline{PN}$.

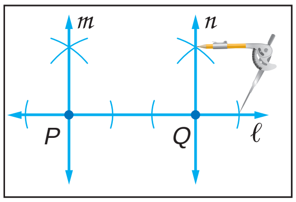
$\overline{MO} \cong \overline{PN}$	Diagonals of a rectangle are ≅.
$MO = PN$	Definition of congruent segments
$6x + 14 = 9x + 5$	Substitution
$14 = 3x + 5$	Subtract 6*x* from each side.
$9 = 3x$	Subtract 5 from each side.
$3 = x$	Divide each side by 3.

Rectangles can be constructed using perpendicular lines.

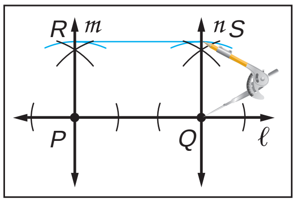
Construction

Rectangle

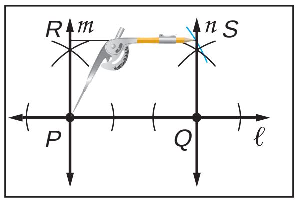
1. Use a straightedge to draw line ℓ. Label a point *P* on ℓ. Place the point at *P* and locate point *Q* on ℓ. Now construct lines perpendicular to ℓ through *P* and through *Q*. Label them m and n.



2. Place the compass point at *P* and mark off a segment on m. Using the same compass setting, place the compass at *Q* and mark a segment on n. Label these points *R* and *S*. Draw $\overline{RS}$.



3. Locate the compass setting that represents *PR* and compare to the setting for *QS*. The measures should be the same.



Study Tip

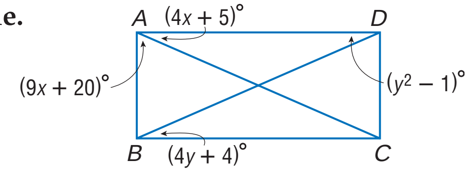
Look Back
To review **constructing perpendicular lines through a point**, see Lesson 3-6.

Example 2 Angles of a Rectangle

ALGEBRA **Quadrilateral *ABCD* is a rectangle.**

a. Find x.

$\angle DAB$ is a right angle, so $m\angle DAB = 90$.



$m\angle DAC + m\angle BAC = m\angle DAB$	Angle Addition Theorem
$4x + 5 + 9x + 20 = 90$	Substitution
$13x + 25 = 90$	Simplify.
$13x = 65$	Subtract 25 from each side.
$x = 5$	Divide each side by 13.

www.geometryonline.com/extra_examples

b. Find y.

Since a rectangle is a parallelogram, opposite sides are parallel. So, alternate interior angles are congruent.

$\angle ADB \cong \angle CBD$	Alternate Interior Angles Theorem
$m\angle ADB = m\angle CBD$	Definition of $\cong$ angles
$y^2 - 1 = 4y + 4$	Substitution
$y^2 - 4y - 5 = 0$	Subtract $4y$ and 4 from each side.
$(y - 5)(y + 1) = 0$	Factor.
$y - 5 = 0 \quad y + 1 = 0$	
$y = 5 \quad y = -1$	Disregard $y = -1$ because it yields angle measures of 0.

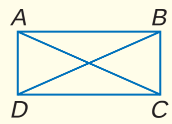
PROVE THAT PARALLELOGRAMS ARE RECTANGLES

The converse of Theorem 8.13 is also true.

Theorem 8.14

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

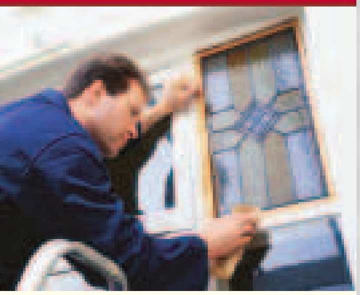
Abbreviation: *If diagonals of ▱ are ≅, ▱ is a rectangle.*



$\overline{AC} \cong \overline{BD}$

You will prove Theorem 8.14 in Exercise 41.

More About. . .



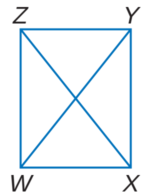
Windows

It is important to square the window frame because over time the opening may have become "out-of-square." If the window is not properly situated in the framed opening, air and moisture can leak through cracks.

Source: www.supersealwindows.com/guide/measurement

Example 3 Diagonals of a Parallelogram

WINDOWS Trent is building a tree house for his younger brother. He has measured the window opening to be sure that the opposite sides are congruent. He measures the diagonals to make sure that they are congruent. This is called *squaring* the frame. How does he know that the corners are 90° angles?



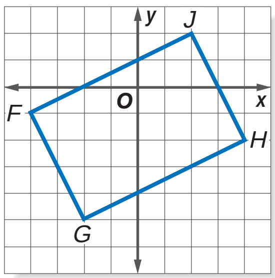
First draw a diagram and label the vertices. We know that $\overline{WX} \cong \overline{ZY}$, $\overline{XY} \cong \overline{WZ}$, and $\overline{WY} \cong \overline{XZ}$.

Because $\overline{WX} \cong \overline{ZY}$ and $\overline{XY} \cong \overline{WZ}$, $WXYZ$ is a parallelogram.

$\overline{XZ}$ and $\overline{WY}$ are diagonals and they are congruent. A parallelogram with congruent diagonals is a rectangle. So, the corners are 90° angles.

Example 4 Rectangle on a Coordinate Plane

COORDINATE GEOMETRY Quadrilateral $FGHJ$ has vertices $F(-4, -1)$, $G(-2, -5)$, $H(4, -2)$, and $J(2, 2)$. Determine whether $FGHJ$ is a rectangle.



Method 1: Use the Slope Formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, to see if consecutive sides are perpendicular.

slope of $\overline{FJ} = \frac{2 - (-1)}{2 - (-4)}$ or $\frac{1}{2}$

$\text{slope of } \overline{GH} = \frac{-2-(-5)}{4-(-2)} \text{ or } \frac{1}{2}$

$\text{slope of } \overline{FG} = \frac{-5-(-1)}{-2-(-4)} \text{ or } -2$

$\text{slope of } \overline{JH} = \frac{2-(-2)}{2-4} \text{ or } -2$

Because $\overline{FJ} \parallel \overline{GH}$ and $\overline{FG} \parallel \overline{JH}$, quadrilateral $FGHJ$ is a parallelogram.

The product of the slopes of consecutive sides is -1. This means that $\overline{FJ} \perp \overline{FG}$, $\overline{FJ} \perp \overline{JH}$, $\overline{JH} \perp \overline{GH}$, and $\overline{FG} \perp \overline{GH}$. The perpendicular segments create four right angles. Therefore, by definition $FGHJ$ is a rectangle.

Method 2: Use the Distance Formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, to determine whether opposite sides are congruent.

First, we must show that quadrilateral $FGHJ$ is a parallelogram.

$$FJ = \sqrt{(-4-2)^2 + (-1-2)^2} = \sqrt{36+9} = \sqrt{45}$$

$$GH = \sqrt{(-2-4)^2 + [-5-(-2)]^2} = \sqrt{36+9} = \sqrt{45}$$

$$FG = \sqrt{[-4-(-2)]^2 + [-1-(-5)]^2} = \sqrt{4+16} = \sqrt{20}$$

$$JH = \sqrt{(2-4)^2 + [2-(-2)]^2} = \sqrt{4+16} = \sqrt{20}$$

Since each pair of opposite sides of the quadrilateral have the same measure, they are congruent. Quadrilateral $FGHJ$ is a parallelogram.

$$FH = \sqrt{(-4-4)^2 + [-1-(-2)]^2} = \sqrt{64+1} = \sqrt{65}$$

$$GJ = \sqrt{(-2-2)^2 + (-5-2)^2} = \sqrt{16+49} = \sqrt{65}$$

The length of each diagonal is $\sqrt{65}$. Since the diagonals are congruent, $FGHJ$ is a rectangle by Theorem 8.14.

Check for Understanding

Concept Check

1. How can you determine whether a parallelogram is a rectangle?

2. **OPEN ENDED** Draw two congruent right triangles with a common hypotenuse. Do the legs form a rectangle?

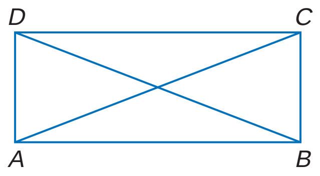
3. **FIND THE ERROR** McKenna and Consuelo are defining a rectangle for an assignment.

McKenna	Consuelo
A rectangle is a parallelogram with one right angle.	A rectangle has a pair of parallel opposite sides and a right angle.

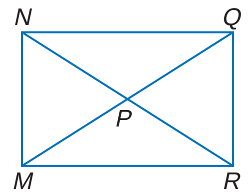
Who is correct? Explain.

Guided Practice

4. **ALGEBRA** *ABCD* is a rectangle. If $AC = 30 - x$ and $BD = 4x - 60$, find x.



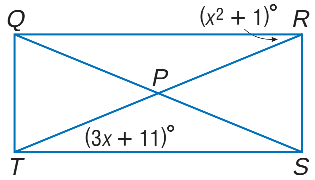
5. **ALGEBRA** *MNQR* is a rectangle. If $NR = 2x + 10$ and $NP = 2x - 30$, find *MP*.



ALGEBRA **Quadrilateral *QRST* is a rectangle. Find each measure or value.**

6. x

7. $m\angle RPS$



8. **COORDINATE GEOMETRY** Quadrilateral *EFGH* has vertices $E(-4, -3)$, $F(3, -1)$, $G(2, 3)$, and $H(-5, 1)$. Determine whether *EFGH* is a rectangle.

Application

9. **FRAMING** Mrs. Walker has a rectangular picture that is 12 inches by 48 inches. Because this is not a standard size, a special frame must be built. What can the framer do to guarantee that the frame is a rectangle? Justify your reasoning.

Practice and Apply

Homework Help

For Exercises	See Examples
10–15, 36–37	1
16–24	2
25–26, 35, 38–46	3
27–34	4

Extra Practice
See page 770.

ALGEBRA **Quadrilateral *JKMN* is a rectangle.**

10. If $NQ = 5x - 3$ and $QM = 4x + 6$, find *NK*.

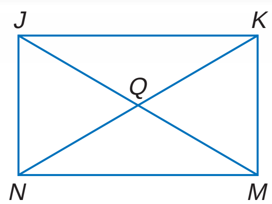
11. If $NQ = 2x + 3$ and $QK = 5x - 9$, find *JQ*.

12. If $NM = 8x - 14$ and $JK = x^2 + 1$, find *JK*.

13. If $m\angle NJM = 2x - 3$ and $m\angle KJM = x + 5$, find x.

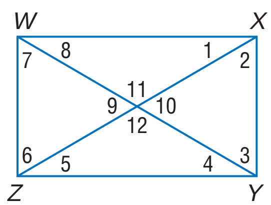
14. If $m\angle NKM = x^2 + 4$ and $m\angle KNM = x + 30$, find $m\angle JKN$.

15. If $m\angle JKN = 2x^2 + 2$ and $m\angle NKM = 14x$, find x.



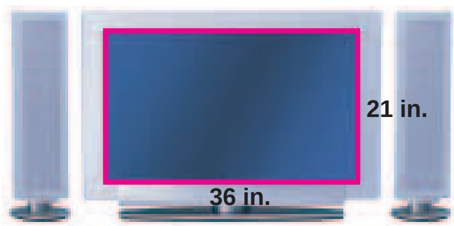
***WXYZ* is a rectangle. Find each measure if $m\angle 1 = 30$.**

16. $m\angle 1$	17. $m\angle 2$	18. $m\angle 3$
19. $m\angle 4$	20. $m\angle 5$	21. $m\angle 6$
22. $m\angle 7$	23. $m\angle 8$	24. $m\angle 9$



25. **PATIOS** A contractor has been hired to pour a rectangular concrete patio. How can he be sure that the frame in which to pour the concrete is rectangular?

26. **TELEVISION** Television screens are measured on the diagonal. What is the measure of the diagonal of this screen?



Zenith Electronics Corp./AP/Wide World Photos

COORDINATE GEOMETRY **Determine whether *DFGH* is a rectangle given each set of vertices. Justify your answer.**

27. $D(9, -1), F(9, 5), G(-6, 5), H(-6, 1)$

28. $D(6, 2), F(8, -1), G(10, 6), H(12, 3)$

29. $D(-4, -3), F(-5, 8), G(6, 9), H(7, -2)$

COORDINATE GEOMETRY **The vertices of *WXYZ* are $W(2, 4)$, $X(-2, 0)$, $Y(-1, -7)$, and $Z(9, 3)$.**

30. Find *WY* and *XZ*.

31. Find the coordinates of the midpoints of $\overline{WY}$ and $\overline{XZ}$.

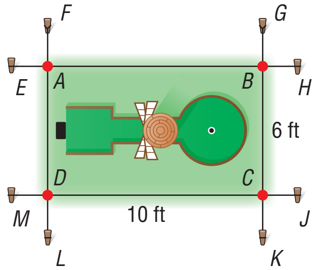
32. Is *WXYZ* a rectangle? Explain.

COORDINATE GEOMETRY **The vertices of parallelogram *ABCD* are $A(-4, -4)$, $B(2, -1)$, $C(0, 3)$, and $D(-6, 0)$.**

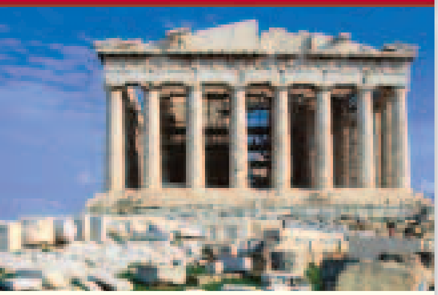
33. Determine whether *ABCD* is a rectangle.

34. If *ABCD* is a rectangle and *E, F, G,* and *H* are midpoints of its sides, what can you conclude about *EFGH*?

35. **MINIATURE GOLF** The windmill section of a miniature golf course will be a rectangle 10 feet long and 6 feet wide. Suppose the contractor placed stakes and strings to mark the boundaries with the corners at *A, B, C* and *D*. The contractor measured $\overline{BD}$ and $\overline{AC}$ and found that $AC > BD$. Describe where to move the stakes *L* and *K* to make *ABCD* a rectangle. Explain.



More About. . .



Golden Rectangles

The Parthenon in ancient Greece is an example of how the golden rectangle was applied to architecture. The ratio of the length to the height is the golden ratio.

Source: www.enc.org

GOLDEN RECTANGLES **For Exercises 36 and 37, use the following information.**
Many artists have used *golden rectangles* in their work. In a golden rectangle, the ratio of the length to the width is about 1.618. This ratio is known as the *golden ratio.*

36. A rectangle has dimensions of 19.42 feet and 12.01 feet. Determine if the rectangle is a golden rectangle. Then find the length of the diagonal.

37. **RESEARCH** Use the Internet or other sources to find examples of golden rectangles.

38. What are the minimal requirements to justify that a parallelogram is a rectangle?

39. Draw a counterexample to the statement *If the diagonals are congruent, the quadrilateral is a rectangle.*

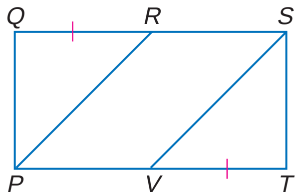
PROOF **Write a two-column proof.**

40. Theorem 8.13

41. Theorem 8.14

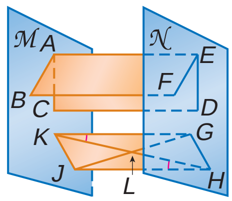
42. Given: *PQST* is a rectangle.
$\overline{QR} \cong \overline{VT}$

Prove: $\overline{PR} \cong \overline{VS}$



43. Given: *DEAC* and *FEAB* are rectangles.
$\angle GKH \cong \angle JHK$
$\overline{GJ}$ and $\overline{HK}$ intersect at *L*.

Prove: *GHJK* is a parallelogram.



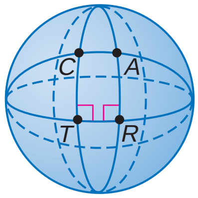
44. **CRITICAL THINKING** Using four of the twelve points as corners, how many rectangles can be drawn?

www.geometryonline.com/self_check_quiz

Izzet Keribar/Lonely Planet Images

CONTENTS

SPHERICAL GEOMETRY The figure shows a *Saccheri quadrilateral* on a sphere. Note that it has four sides with $\overline{CT} \perp \overline{TR}$, $\overline{AR} \perp \overline{TR}$, and $\overline{CT} \cong \overline{AR}$.



45. Is $\overline{CT}$ parallel to $\overline{AR}$? Explain.

46. How does AC compare to TR?

47. Can a rectangle exist in spherical geometry? Explain.

48. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

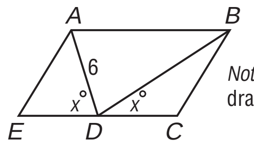
How are rectangles used in tennis?

Include the following in your answer:
- the number of rectangles on one side of a tennis court, and
- a method to ensure the lines on the court are parallel

Standardized Test Practice

49. In the figure, $\overline{AB} \parallel \overline{CE}$. If $DA = 6$, what is DB?

(A) 6 (B) 7
(C) 8 (D) 9



Note: Figure not drawn to scale

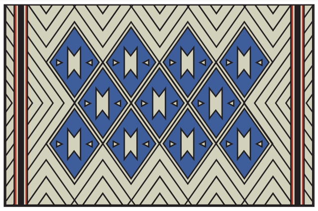
50. **ALGEBRA** A rectangular playground is surrounded by an 80-foot long fence. One side of the playground is 10 feet longer than the other. Which of the following equations could be used to find s, the shorter side of the playground?

(A) $10s + s = 80$ (B) $4s + 10 = 80$
(C) $s(s + 10) = 80$ (D) $2(s + 10) + 2s = 80$

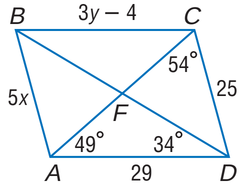
Maintain Your Skills

Mixed Review

51. **TEXTILE ARTS** The Navajo people are well known for their skill in weaving. The design at the right, known as the Eye-Dazzler, became popular with Navajo weavers in the 1880s. How many parallelograms, not including rectangles, are in the pattern? *(Lesson 8-3)*



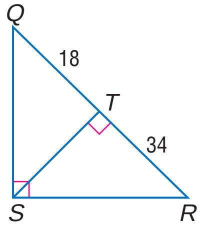
For Exercises 52–57, use ▱$ABCD$. Find each measure or value. *(Lesson 8-2)*



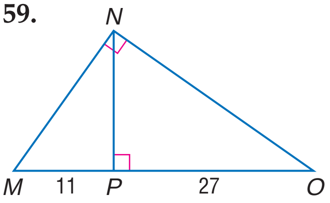
52. $m\angle AFD$ **53.** $m\angle CDF$
54. $m\angle FBC$ **55.** $m\angle BCF$
56. y **57.** x

Find the measure of the altitude of each triangle. *(Lesson 7-2)*

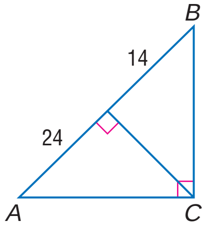
58.



59.



60.



Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the distance between each pair of points.**
*(To review the **Distance Formula**, see Lesson 1-4.)*

61. $(1, -2), (-3, 1)$ **62.** $(-5, 9), (5, 12)$ **63.** $(1, 4), (22, 24)$

8-5 Rhombi and Squares

What You'll Learn

- Recognize and apply the properties of rhombi.
- Recognize and apply the properties of squares.

How can you ride a bicycle with square wheels?

Most bicycles have round wheels, but Professor Stan Wagon at Macalester College in St. Paul, Minnesota, developed one with square wheels. There are two front wheels so the rider can balance without turning the handlebars. Riding over a specially curved road means there is a smooth ride.

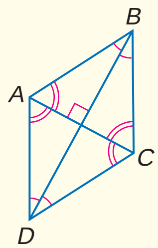
Vocabulary

- rhombus
- square

PROPERTIES OF RHOMBI A square is a special type of parallelogram called a rhombus. A **rhombus** is a quadrilateral with all four sides congruent. Since opposite sides are congruent, the rhombus is a parallelogram, and all of the properties of parallelograms can be applied to rhombi. There are three other characteristics of rhombi described in the following theorems.

Key Concept — Rhombus

	Theorem	Example
8.15	The diagonals of a rhombus are perpendicular.	$\overline{AC} \perp \overline{BD}$
8.16	If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (Converse of Theorem 8.15)	If $\overline{BD} \perp \overline{AC}$, then ▱$ABCD$ is a rhombus.
8.17	Each diagonal of a rhombus bisects a pair of opposite angles.	$\angle DAC \cong \angle BAC \cong \angle DCA \cong \angle BCA$ $\angle ABD \cong \angle CBD \cong \angle ADB \cong \angle CDB$



You will prove Theorems 8.16 and 8.17 in Exercises 35 and 36, respectively.

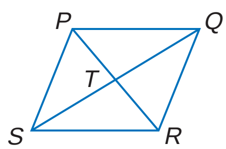
Study Tip

Proof
Since a rhombus has four congruent sides, one diagonal separates the rhombus into two congruent isosceles triangles. Drawing two diagonals separates the rhombus into four congruent right triangles.

Example 1 Proof of Theorem 8.15

Given: $PQRS$ is a rhombus.

Prove: $\overline{PR} \perp \overline{SQ}$



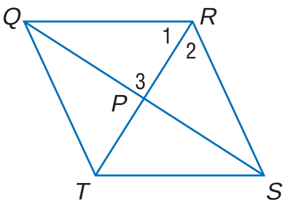
Proof:
By the definition of a rhombus, $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{PS}$. A rhombus is a parallelogram and the diagonals of a parallelogram bisect each other, so $\overline{QS}$ bisects $\overline{PR}$ at T. Thus, $\overline{PT} \cong \overline{RT}$. $\overline{QT} \cong \overline{QT}$ because congruence of segments is reflexive. Thus, $\triangle PQT \cong \triangle RQT$ by SSS. $\angle QTP \cong \angle QTR$ by CPCTC. $\angle QTP$ and $\angle QTR$ also form a linear pair. Two congruent angles that form a linear pair are right angles. $\angle QTP$ is a right angle, so $\overline{PR} \perp \overline{SQ}$ by the definition of perpendicular lines.

Courtesy Professor Stan Wagon/Photo by Deanna Haunsperger

Study Tip

Reading Math
The plural form of rhombus is *rhombi*, pronounced ROM-bye.

Example 2 Measures of a Rhombus



ALGEBRA **Use rhombus *QRST* and the given information to find the value of each variable.**

a. Find y if $m\angle 3 = y^2 - 31$.

$m\angle 3 = 90$ The diagonals of a rhombus are perpendicular.

$y^2 - 31 = 90$ Substitution

$y^2 = 121$ Add 31 to each side.

$y = \pm 11$ Take the square root of each side.

The value of y can be 11 or -11.

b. Find $m\angle TQS$ if $m\angle RST = 56$.

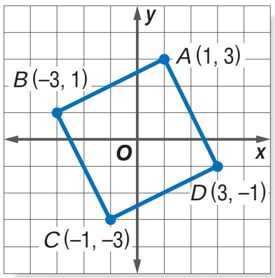
$m\angle TQR = m\angle RST$ Opposite angles are congruent.

$m\angle TQR = 56$ Substitution

The diagonals of a rhombus bisect the angles. So, $m\angle TQS$ is $\frac{1}{2}(56)$ or 28.

PROPERTIES OF SQUARES If a quadrilateral is both a rhombus and a rectangle, then it is a **square**. All of the properties of parallelograms and rectangles can be applied to squares.

Example 3 Squares



COORDINATE GEOMETRY **Determine whether parallelogram *ABCD* is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.**

Explore Plot the vertices on a coordinate plane.

Plan If the diagonals are perpendicular, then *ABCD* is either a rhombus or a square. The diagonals of a rectangle are congruent. If the diagonals are congruent and perpendicular, then *ABCD* is a square.

Solve Use the Distance Formula to compare the lengths of the diagonals.

$$DB = \sqrt{[3-(-3)]^2 + (-1-1)^2} = \sqrt{36+4} = \sqrt{40}$$

$$AC = \sqrt{(1+1)^2 + (3+3)^2} = \sqrt{4+36} = \sqrt{40}$$

Use slope to determine whether the diagonals are perpendicular.

slope of $\overline{DB} = \frac{1-(-1)}{-3-3}$ or $-\frac{1}{3}$ slope of $\overline{AC} = \frac{-3-3}{-1-1}$ or 3

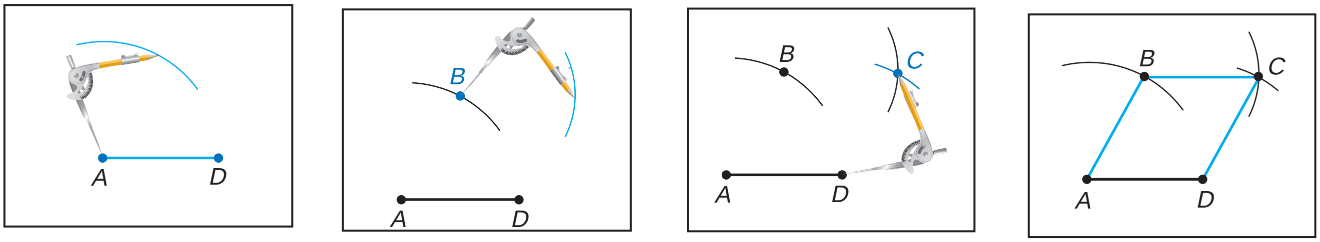
Since the slope of $\overline{AC}$ is the negative reciprocal of the slope of $\overline{DB}$, the diagonals are perpendicular. The lengths of $\overline{DB}$ and $\overline{AC}$ are the same so the diagonals are congruent. *ABCD* is a rhombus, a rectangle, and a square.

Examine The diagonals are congruent and perpendicular so *ABCD* must be a square. You can verify that *ABCD* is a rhombus by finding *AB*, *BC*, *CD*, and *AD*. Then see if two consecutive segments are perpendicular.

Construction

Rhombus

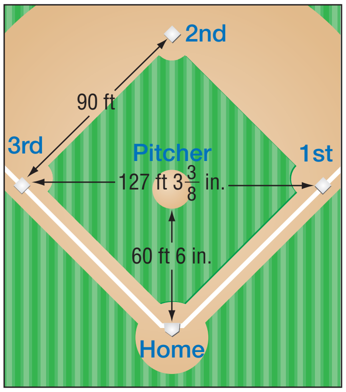
1. Draw any segment $\overline{AD}$. Place the compass point at *A*, open to the width of *AD*, and draw an arc above $\overline{AD}$.
2. Label any point on the arc as *B*. Using the same setting, place the compass at *B*, and draw an arc to the right of *B*.
3. Then place the compass at point *D*, and draw an arc to intersect the arc drawn from point *B*. Label the point of intersection *C*.
4. Use a straightedge to draw $\overline{AB}$, $\overline{BC}$, and $\overline{CD}$.



Conclusion: Since all of the sides are congruent, quadrilateral *ABCD* is a rhombus.

Example 4 *Diagonals of a Square*

BASEBALL **The infield of a baseball diamond is a square, as shown at the right. Is the pitcher's mound located in the center of the infield? Explain.**



Since a square is a parallelogram, the diagonals bisect each other. Since a square is a rhombus, the diagonals are congruent. Therefore, the distance from first base to third base is equal to the distance between home plate and second base. Thus, the distance from home plate to the center of the infield is 127 feet $3\frac{3}{8}$ inches divided by 2 or 63 feet $7\frac{11}{16}$ inches. This distance is longer than the distance from home plate to the pitcher's mound so the pitcher's mound is not located in the center of the field. It is about 3 feet closer to home.

If a quadrilateral is a rhombus or a square, then the following properties are true.

Study Tip

Square and Rhombus

A square is a rhombus, but a rhombus is not necessarily a square.

Concept Summary — *Properties of Rhombi and Squares*

Rhombi

1. A rhombus has all the properties of a parallelogram.
2. All sides are congruent.
3. Diagonals are perpendicular.
4. Diagonals bisect the angles of the rhombus.

Squares

1. A square has all the properties of a parallelogram.
2. A square has all the properties of a rectangle.
3. A square has all the properties of a rhombus.

Check for Understanding

Concept Check

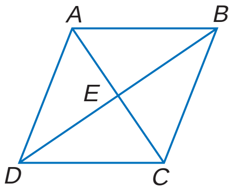
1. **Draw a diagram** to demonstrate the relationship among parallelograms, rectangles, rhombi, and squares.

2. **OPEN ENDED** Draw a quadrilateral that has the characteristics of a rectangle, a rhombus, and a square.

3. **Explain** the difference between a square and a rectangle.

Guided Practice

ALGEBRA **In rhombus $ABCD$, $AB = 2x + 3$ and $BC = 5x$.**

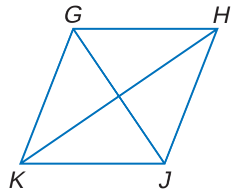


4. Find x.
5. Find AD.
6. Find $m\angle AEB$.
7. Find $m\angle BCD$ if $m\angle ABC = 83.2$.

COORDINATE GEOMETRY **Given each set of vertices, determine whether $\square MNPQ$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain your reasoning.**

8. $M(0, 3)$, $N(-3, 0)$, $P(0, -3)$, $Q(3, 0)$
9. $M(-4, 0)$, $N(-3, 3)$, $P(2, 2)$, $Q(1, -1)$

10. **PROOF** Write a two-column proof.
Given: $\triangle KGH$, $\triangle HJK$, $\triangle GHJ$, and $\triangle JKG$ are isosceles.
Prove: $GHJK$ is a rhombus.



Application

11. **REMODELING** The Steiner family is remodeling their kitchen. Each side of the floor measures 10 feet. What other measurements should be made to determine whether the floor is a square?

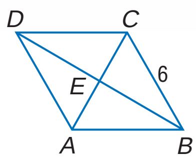
Practice and Apply

Homework Help

For Exercises	See Examples
12–19	2
20–23	3
24–36	4
37–42	1

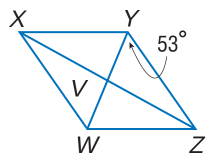
Extra Practice
See page 770.

In rhombus $ABCD$, $m\angle DAB = 2m\angle ADC$ and $CB = 6$.



12. Find $m\angle ACD$.
13. Find $m\angle DAB$.
14. Find DA.
15. Find $m\angle ADB$.

ALGEBRA **Use rhombus $XYZW$ with $m\angle WYZ = 53$, $VW = 3$, $XV = 2a - 2$, and $ZV = \frac{5a + 1}{4}$.**



16. Find $m\angle YZV$.
17. Find $m\angle XYW$.
18. Find XZ.
19. Find XW.

COORDINATE GEOMETRY **Given each set of vertices, determine whether $\square EFGH$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain your reasoning.**

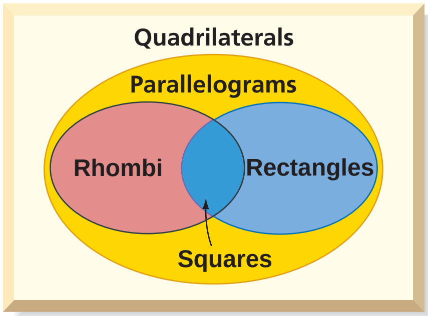
20. $E(1, 10)$, $F(-4, 0)$, $G(7, 2)$, $H(12, 12)$
21. $E(-7, 3)$, $F(-2, 3)$, $G(1, 7)$, $H(-4, 7)$
22. $E(1, 5)$, $F(6, 5)$, $G(6, 10)$, $H(1, 10)$
23. $E(-2, -1)$, $F(-4, 3)$, $G(1, 5)$, $H(3, 1)$

CONSTRUCTION **Construct each figure using a compass and straightedge.**

24. a square with one side 3 centimeters long

25. a square with a diagonal 5 centimeters long

Use the Venn diagram to determine whether each statement is *always*, *sometimes*, or *never* true.



26. A parallelogram is a square.

27. A square is a rhombus.

28. A rectangle is a parallelogram.

29. A rhombus is a rectangle.

30. A rhombus is a square.

31. A square is a rectangle.

More About. . .

Design

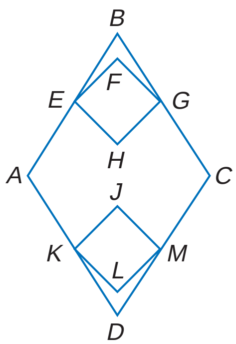
The plant stand is constructed from painted wood and metal. The overall dimensions are $36\frac{1}{2}$ inches tall by $15\frac{3}{4}$ inches wide.

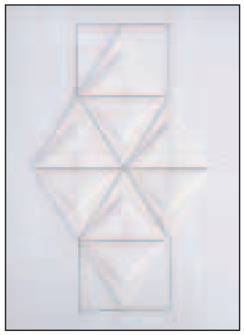
Source: www.metmuseum.org

32. DESIGN Otto Prutscher designed the plant stand at the left in 1903. The base is a square, and the base of each of the five boxes is also a square. Suppose each smaller box is one half as wide as the base. Use the information at the left to find the dimensions of one of the smaller boxes.

33. PERIMETER The diagonals of a rhombus are 12 centimeters and 16 centimeters long. Find the perimeter of the rhombus.

34. ART This piece of art is Dorthea Rockburne's *Egyptian Painting: Scribe*. The diagram shows three of the shapes shown in the piece. Use a ruler or a protractor to determine which type of quadrilateral is represented by each figure.



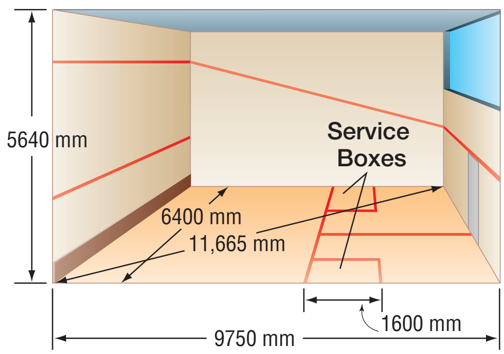


PROOF **Write a paragraph proof for each theorem.**

35. Theorem 8.16

36. Theorem 8.17

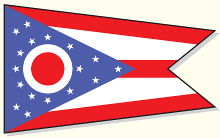
SQUASH **For Exercises 37 and 38, use the diagram of the court for squash, a game similar to racquetball and tennis.**



37. The diagram labels the diagonal as 11,665 millimeters. Is this correct? Explain.

38. The service boxes are squares. Find the length of the diagonal.

More About. . .

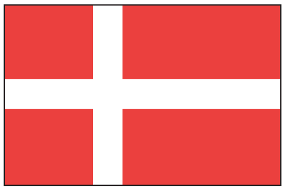


Flags

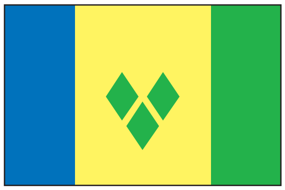
The state of Ohio has the only state flag in the United States that is not rectangular.

Source: *World Almanac*

39. **FLAGS** Study the flags shown below. Use a ruler and protractor to determine if any of the flags contain parallelograms, rectangles, rhombi, or squares.



Denmark



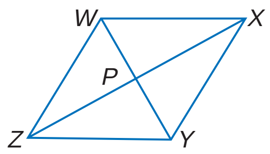
St. Vincent and The Grenadines

Trinidad and Tobago

PROOF **Write a two-column proof.**

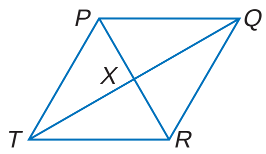
40. **Given:** $\triangle WZY \cong \triangle WXY$, $\triangle WZY$ and $\triangle XYZ$ are isosceles.

Prove: $WXYZ$ is a rhombus.



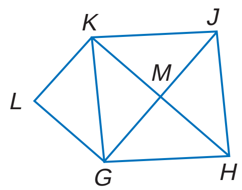
41. **Given:** $\triangle TPX \cong \triangle QPX \cong \triangle QRX \cong \triangle TRX$

Prove: $TPQR$ is a rhombus.



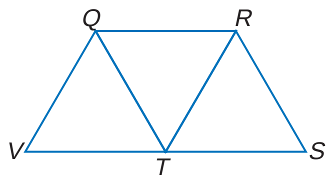
42. **Given:** $\triangle LGK \cong \triangle MJK$; $GHJK$ is a parallelogram.

Prove: $GHJK$ is a rhombus.



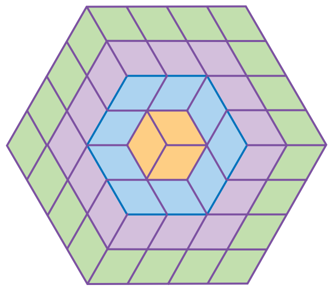
43. **Given:** $QRST$ and $QRTV$ are rhombi.

Prove: $\triangle QRT$ is equilateral.



44. **CRITICAL THINKING** The pattern at the right is a series of rhombi that continue to form a hexagon that increases in size. Copy and complete the table.

Hexagon	Number of rhombi
1	3
2	12
3	27
4	48
5	
6	
x	



45. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you ride a bicycle with square wheels?

Include the following in your answer:

- difference between squares and rhombi, and
- how nonsquare rhombus-shaped wheels would work with the curved road.

Standardized Test Practice

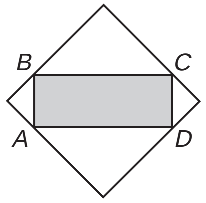
46. Points A, B, C, and D are on a square. The area of the square is 36 square units. Which of the following statements is true?

(A) The perimeter of rectangle $ABCD$ is greater than 24 units.

(B) The perimeter of rectangle $ABCD$ is less than 24 units.

(C) The perimeter of rectangle $ABCD$ is equal to 24 units.

(D) The perimeter of rectangle $ABCD$ cannot be determined from the information given.



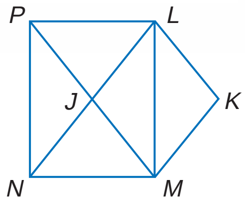
47. **ALGEBRA** For all integers $x \neq 2$, let $<x> = \frac{1+x}{x-2}$. Which of the following has the greatest value?

(A) $<0>$ (B) $<1>$ (C) $<3>$ (D) $<4>$

Maintain Your Skills

Mixed Review

ALGEBRA **Use rectangle $LMNP$, parallelogram $LKMJ$, and the given information to solve each problem.** *(Lesson 8-4)*

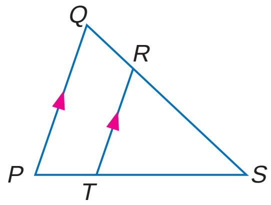


48. If $LN = 10$, $LJ = 2x + 1$, and $PJ = 3x - 1$, find x.
49. If $m\angle PLK = 110$, find $m\angle LKM$.
50. If $m\angle MJN = 35$, find $m\angle MPN$.
51. If $MK = 6x$, $KL = 3x + 2y$, and $JN = 14 - x$, find x and y.
52. If $m\angle LMP = m\angle PMN$, find $m\angle PJL$.

COORDINATE GEOMETRY **Determine whether the coordinates of the vertices of the quadrilateral form a parallelogram. Use the method indicated.** *(Lesson 8-3)*

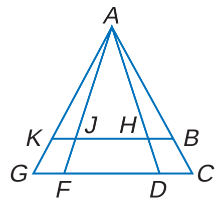
53. $P(0, 2)$, $Q(6, 4)$, $R(4, 0)$, $S(-2, -2)$; Distance Formula
54. $F(1, -1)$, $G(-4, 1)$, $H(-3, 4)$, $J(2, 1)$; Distance Formula
55. $K(-3, -7)$, $L(3, 2)$, $M(1, 7)$, $N(-3, 1)$; Slope Formula
56. $A(-4, -1)$, $B(-2, -5)$, $C(1, 7)$, $D(3, 3)$; Slope Formula

Refer to $\triangle PQS$. *(Lesson 6-4)*



57. If $RT = 16$, $QP = 24$, and $ST = 9$, find PS.
58. If $PT = y - 3$, $PS = y + 2$, $RS = 12$, and $QS = 16$, solve for y.
59. If $RT = 15$, $QP = 21$, and $PT = 8$, find TS.

Refer to the figure. *(Lesson 4-6)*



60. If $\overline{AG} \cong \overline{AC}$, name two congruent angles.
61. If $\overline{AJ} \cong \overline{AH}$, name two congruent angles.
62. If $\angle AFD \cong \angle ADF$, name two congruent segments.
63. If $\angle AKB \cong \angle ABK$, name two congruent segments.

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation.**

*(To review **solving equations**, see pages 737 and 738.)*

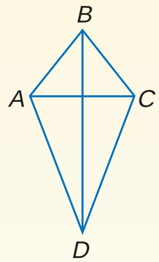
64. $\frac{1}{2}(8x - 6x - 7) = 5$
65. $\frac{1}{2}(7x + 3x + 1) = 12.5$
66. $\frac{1}{2}(4x + 6 + 2x + 13) = 15.5$
67. $\frac{1}{2}(7x - 2 + 3x + 3) = 25.5$

Geometry Activity

A Follow-Up to Lesson 8-5

Kites

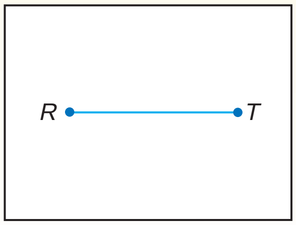
A **kite** is a quadrilateral with exactly two distinct pairs of adjacent congruent sides. In kite $ABCD$, diagonal $\overline{BD}$ separates the kite into two congruent triangles. Diagonal $\overline{AC}$ separates the kite into two noncongruent isosceles triangles.



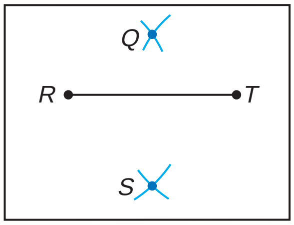
Activity

Construct a kite $QRST$.

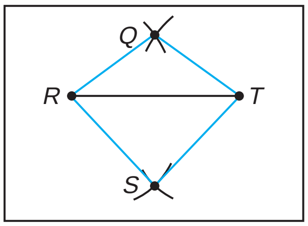
1. Draw $\overline{RT}$.



2. Choose a compass setting greater than $\frac{1}{2}\,\overline{RT}$. Place the compass at point R and draw an arc above $\overline{RT}$. Then without changing the compass setting, move the compass to point T and draw an arc that intersects the first one. Label the intersection point Q. Increase the compass setting. Place the compass at R and draw an arc below $\overline{RT}$. Then, without changing the compass setting, draw an arc from point T to intersect the other arc. Label the intersection point S.



3. Draw $QRST$.



Model

1. Draw $\overline{QS}$ in kite $QRST$. Use a protractor to measure the angles formed by the intersection of $\overline{QS}$ and $\overline{RT}$.
2. Measure the interior angles of kite $QRST$. Are any congruent?
3. Label the intersection of $\overline{QS}$ and $\overline{RT}$ as point N. Find the lengths of $\overline{QN}$, $\overline{NS}$, $\overline{TN}$, and $\overline{NR}$. How are they related?
4. How many pairs of congruent triangles can be found in kite $QRST$?
5. Construct another kite $JKLM$. Repeat Exercises 1–4.

Analyze

6. Use your observations and measurements of kites $QRST$ and $JKLM$ to make conjectures about the angles, sides, and diagonals of kites.

8-6 Trapezoids

What You'll Learn

- Recognize and apply the properties of trapezoids.
- Solve problems involving the medians of trapezoids.

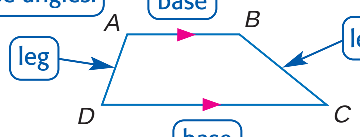
How are trapezoids used in architecture?

The Washington Monument in Washington, D.C., is an obelisk made of white marble. The width of the base is longer than the width at the top. Each face of the monument is an example of a trapezoid.

Vocabulary

- trapezoid
- isosceles trapezoid
- median

PROPERTIES OF TRAPEZOIDS A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called *bases*. The *base angles* are formed by the base and one of the legs. The nonparallel sides are called *legs*.



If the legs are congruent, then the trapezoid is an **isosceles trapezoid**. Theorems 8.18 and 8.19 describe two characteristics of isosceles trapezoids.

Study Tip

Isosceles Trapezoid
If you extend the legs of an isosceles trapezoid until they meet, you will have an isosceles triangle. Recall that the base angles of an isosceles triangle are congruent.

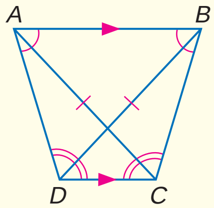
Theorems

8.18 Both pairs of base angles of an isosceles trapezoid are congruent.

8.19 The diagonals of an isosceles trapezoid are congruent.

Example:
$\angle DAB \cong \angle CBA$
$\angle ADC \cong \angle BCD$

$\overline{AC} \cong \overline{BD}$



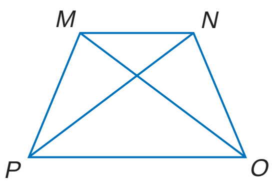
You will prove Theorem 8.18 in Exercise 36.

Example 1 Proof of Theorem 8.19

Write a flow proof of Theorem 8.19.

Given: *MNOP* is an isosceles trapezoid.

Prove: $\overline{MO} \cong \overline{NP}$

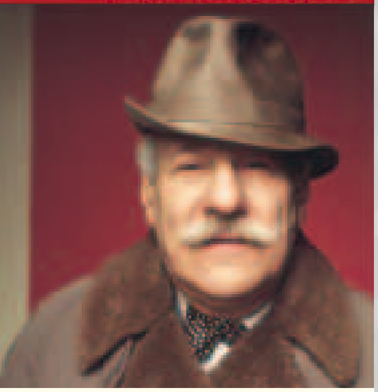


Proof:

- *MNOP* is an isosceles trapezoid. (Given) → $\overline{MP} \cong \overline{NO}$ (Def. of isos. trapezoid)
- *MNOP* is an isosceles trapezoid. (Given) → $\angle MPO \cong \angle NOP$ (Base ∠s of isos. trap. are ≅.)
- $\overline{PO} \cong \overline{PO}$ (Reflexive Property)
- $\overline{MP} \cong \overline{NO}$, $\angle MPO \cong \angle NOP$, $\overline{PO} \cong \overline{PO}$ → $\triangle MPO \cong \triangle NOP$ (SAS)
- $\triangle MPO \cong \triangle NOP$ → $\overline{MO} \cong \overline{NP}$ (CPCTC)

Example 2 Identify Isosceles Trapezoids

More About...



Art

Barnett Newman designed this piece to be 50% larger. This piece was built for an exhibition in Japan but it could not be built as large as the artist wanted because of size limitations on cargo from New York to Japan.

Source: www.sfmoma.org

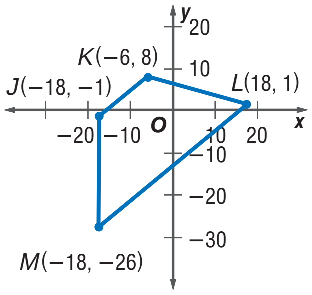
ART The sculpture pictured is *Zim Zum I* by Barnett Newman. The walls are connected at right angles. In perspective, the rectangular panels appear to be trapezoids. Use a ruler and protractor to determine if the images of the front panels are isosceles trapezoids. Explain.

The panel on the left is an isosceles trapezoid. The bases are parallel and are different lengths. The legs are not parallel and they are the same length.

The panel on the right is not an isosceles trapezoid. Each side is a different length.

Example 3 Identify Trapezoids

COORDINATE GEOMETRY *JKLM* is a quadrilateral with vertices $J(-18, -1)$, $K(-6, 8)$, $L(18, 1)$, and $M(-18, -26)$.



a. Verify that *JKLM* is a trapezoid.

A quadrilateral is a trapezoid if exactly one pair of opposite sides are parallel. Use the Slope Formula.

$$\text{slope of } \overline{JK} = \frac{-1-8}{-18-(-6)} = \frac{-9}{-12} \text{ or } \frac{3}{4}$$

$$\text{slope of } \overline{ML} = \frac{1-(-26)}{18-(-18)} = \frac{27}{36} \text{ or } \frac{3}{4}$$

$$\text{slope of } \overline{JM} = \frac{-1-(-26)}{-18-(-18)} = \frac{25}{0} \text{ or undefined}$$

$$\text{slope of } \overline{KL} = \frac{1-8}{18-(-6)} = \frac{-7}{24}$$

Exactly one pair of opposite sides are parallel, $\overline{JK}$ and $\overline{ML}$. So, *JKLM* is a trapezoid.

b. Determine whether *JKLM* is an isosceles trapezoid. Explain.

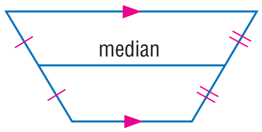
First use the Distance Formula to show that the legs are congruent.

$$JM = \sqrt{[-18-(-18)]^2 + [-1-(-26)]^2} = \sqrt{0+625} = \sqrt{625} \text{ or } 25$$

$$KL = \sqrt{(-6-18)^2 + (8-1)^2} = \sqrt{576+49} = \sqrt{625} \text{ or } 25$$

Since the legs are congruent, *JKLM* is an isosceles trapezoid.

MEDIANS OF TRAPEZOIDS



The segment that joins midpoints of the legs of a trapezoid is the **median**. The median of a trapezoid can also be called a *midsegment*. Recall from Lesson 6-4 that the midsegment of a triangle is the segment joining the midpoints of two sides. The median of a trapezoid has the same properties as the midsegment of a triangle. You can construct the median of a trapezoid using a compass and a straightedge.

Study Tip

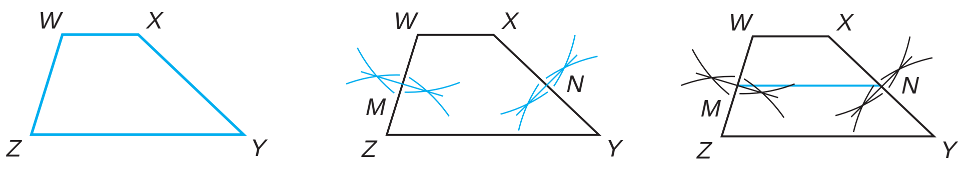
Reading Math
The word *median* means *middle*. The median of a trapezoid is the segment that is parallel to and equidistant from each base.

Geometry Activity

Median of a Trapezoid

Model

1. Draw trapezoid $WXYZ$ with legs $\overline{XY}$ and $\overline{WZ}$.
2. Construct the perpendicular bisectors of $\overline{XY}$ and $\overline{WZ}$. Label the midpoints.
3. Draw $\overline{MN}$.



Analyze

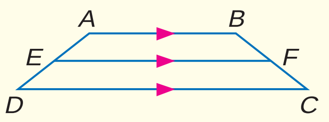
1. Measure $\overline{WX}$, $\overline{ZY}$, and $\overline{MN}$ to the nearest millimeter.
2. Make a conjecture based on your observations.

The results of the Geometry Activity suggest Theorem 8.20.

Theorem 8.20

The median of a trapezoid is parallel to the bases, and its measure is one-half the sum of the measures of the bases.

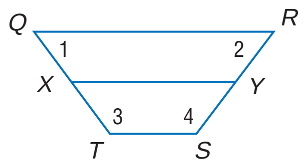
Example: $EF = \frac{1}{2}(AB + DC)$



Example 4 Median of a Trapezoid

ALGEBRA **$QRST$ is an isosceles trapezoid with median $\overline{XY}$.**

a. Find TS if $QR = 22$ and $XY = 15$.



$XY = \frac{1}{2}(QR + TS)$ Theorem 8.20

$15 = \frac{1}{2}(22 + TS)$ Substitution

$30 = 22 + TS$ Multiply each side by 2.

$8 = TS$ Subtract 22 from each side.

b. Find $m\angle 1$, $m\angle 2$, $m\angle 3$, and $m\angle 4$ if $m\angle 1 = 4a - 10$ and $m\angle 3 = 3a + 32.5$.

Since $\overline{QR} \parallel \overline{TS}$, $\angle 1$ and $\angle 3$ are supplementary. Because this is an isosceles trapezoid, $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$.

$m\angle 1 + m\angle 3 = 180$ Consecutive Interior Angles Theorem

$4a - 10 + 3a + 32.5 = 180$ Substitution

$7a + 22.5 = 180$ Combine like terms.

$7a = 157.5$ Subtract 22.5 from each side.

$a = 22.5$ Divide each side by 7.

If $a = 22.5$, then $m\angle 1 = 80$ and $m\angle 3 = 100$.

Because $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$, $m\angle 2 = 80$ and $m\angle 4 = 100$.

Check for Understanding

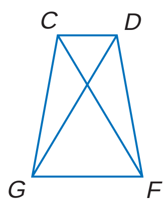
Concept Check

1. **List** the minimum requirements to show that a quadrilateral is a trapezoid.
2. **Make a chart** comparing the characteristics of the diagonals of a trapezoid, a rectangle, a square, and a rhombus. (*Hint:* Use the types of quadrilaterals as column headings and the properties of diagonals as row headings.)
3. **OPEN ENDED** Draw a trapezoid and an isosceles trapezoid. Draw the median for each. Is the median parallel to the bases in both trapezoids?

Guided Practice

COORDINATE GEOMETRY ***QRST*** **is a quadrilateral with vertices** $Q(-3, 2)$, $R(-1, 6)$, $S(4, 6)$, $T(6, 2)$.

4. Verify that *QRST* is a trapezoid.
5. Determine whether *QRST* is an isosceles trapezoid. Explain.
6. **PROOF** *CDFG* is an isosceles trapezoid with bases $\overline{CD}$ and $\overline{FG}$. Write a flow proof to prove $\angle DGF \cong \angle CFG$.



7. **ALGEBRA** *EFGH* is an isosceles trapezoid with median $\overline{YZ}$. If $EF = 3x + 8$, $HG = 4x - 10$, and $YZ = 13$, find x.

Application

8. **PHOTOGRAPHY** Photographs can show a building in a perspective that makes it appear to be a different shape. Identify the types of quadrilaterals in the photograph.

Practice and Apply

Homework Help

For Exercises	See Examples
9–12, 23–32	3
13–19, 39	4
20–22, 38	2
33–37	1

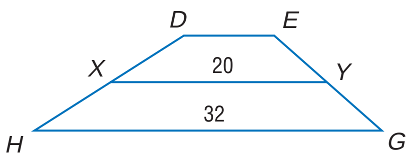
Extra Practice
See page 770.

COORDINATE GEOMETRY **For each quadrilateral whose vertices are given,**
a. verify that the quadrilateral is a trapezoid, and
b. determine whether the figure is an isosceles trapezoid.

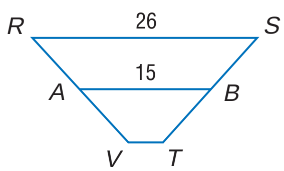
9. $A(-3, 3)$, $B(-4, -1)$, $C(5, -1)$, $D(2, 3)$
10. $G(-5, -4)$, $H(5, 4)$, $J(0, 5)$, $K(-5, 1)$
11. $C(-1, 1)$, $D(-5, -3)$, $E(-4, -10)$, $F(6, 0)$
12. $Q(-12, 1)$, $R(-9, 4)$, $S(-4, 3)$, $T(-11, -4)$

ALGEBRA **Find the missing measure(s) for the given trapezoid.**

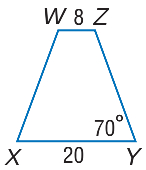
13. For trapezoid *DEGH*, *X* and *Y* are midpoints of the legs. Find *DE*.



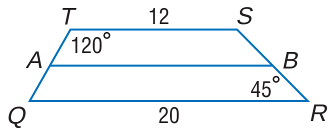
14. For trapezoid *RSTV*, *A* and *B* are midpoints of the legs. Find *VT*.



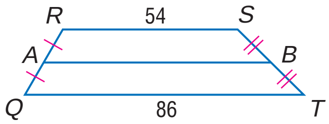
15. For isosceles trapezoid $XYZW$, find the length of the median, $m\angle W$, and $m\angle Z$.



16. For trapezoid $QRST$, A and B are midpoints of the legs. Find AB, $m\angle Q$, and $m\angle S$.



For Exercises 17 and 18, use trapezoid *QRST*.



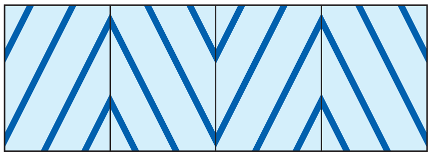
17. Let $\overline{GH}$ be the median of $RSBA$. Find GH.

18. Let $\overline{JK}$ be the median of $ABTQ$. Find JK.

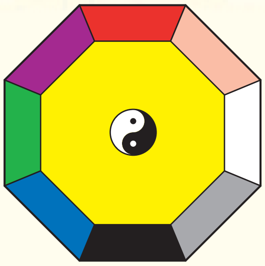
19. **ALGEBRA** $JKLM$ is a trapezoid with $\overline{JK} \parallel \overline{LM}$ and median $\overline{RP}$. Find RP if $JK = 2(x + 3)$, $RP = 5 + x$, and $ML = \frac{1}{2}x - 1$.

20. **DESIGN** The bagua is a tool used in Feng Shui design. This bagua consists of two regular octagons centered around a yin-yang symbol. How can you determine the type of quadrilaterals in the bagua?

21. **SEWING** Madison is making a valance for a window treatment. She is using striped fabric cut on the bias, or diagonal, to create a chevron pattern. Identify the polygons formed in the fabric.



More About. . .



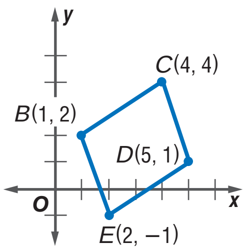
Design

Feng Shui is an ancient Chinese theory of design. The goal is to create spaces that enhance creativity and balance.

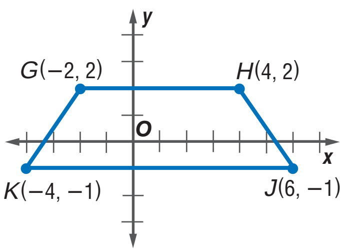
Source: www.fengshui2000.com

COORDINATE GEOMETRY **Determine whether each figure is a *trapezoid*, a *parallelogram*, a *square*, a *rhombus*, or a *quadrilateral*. Choose the most specific term. Explain.**

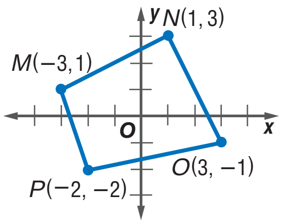
22.



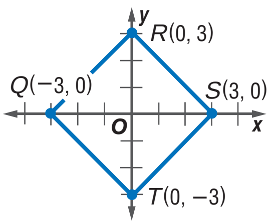
23.



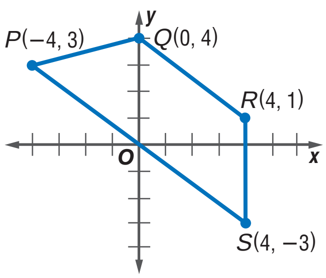
24.



25.



COORDINATE GEOMETRY **For Exercises 26–28, refer to quadrilateral *PQRS*.**

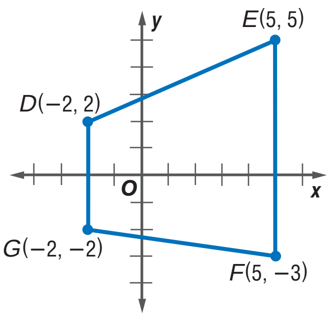


26. Determine whether the figure is a trapezoid. If so, is it isosceles? Explain.

27. Find the coordinates of the midpoints of $\overline{PQ}$ and $\overline{RS}$, and label them A and B.

28. Find AB without using the Distance Formula.

COORDINATE GEOMETRY **For Exercises 29–31, refer to quadrilateral *DEFG*.**



29. Determine whether the figure is a trapezoid. If so, is it isosceles? Explain.

30. Find the coordinates of the midpoints of $\overline{DE}$ and $\overline{GF}$, and label them W and V.

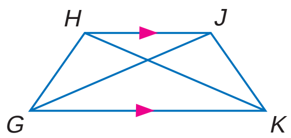
31. Find WV without using the Distance Formula.

WebQuest

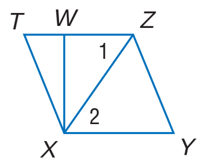
You can use a map of Seattle to locate and draw a quadrilateral that will help you begin to find the hidden treasure. Visit www.geometryonline.com/webquest to continue work on your WebQuest project.

PROOF **Write a flow proof.**

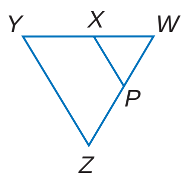
32. **Given:** $\overline{HJ} \parallel \overline{GK}$, $\triangle HGK \cong \triangle JKG$
Prove: $GHJK$ is an isosceles trapezoid.



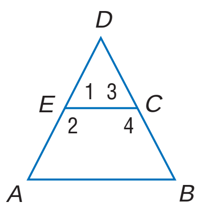
33. **Given:** $\triangle TZX \cong \triangle YXZ$
Prove: $XYZW$ is a trapezoid.



34. **Given:** $ZYXP$ is an isosceles trapezoid.
Prove: $\triangle PWX$ is isosceles.



35. **Given:** E and C are midpoints of $\overline{AD}$ and $\overline{DB}$. $\overline{AD} \cong \overline{DB}$
Prove: $ABCE$ is an isosceles trapezoid.



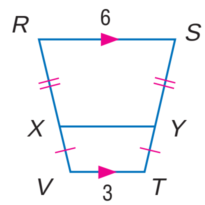
36. Write a paragraph proof of Theorem 8.18.

CONSTRUCTION **Use a compass and straightedge to construct each figure.**

37. an isosceles trapezoid

38. trapezoid with a median 2 centimeters long

39. **CRITICAL THINKING** In $RSTV$, $RS = 6$, $VT = 3$, and RX is twice the length of XV. Find XY.



40. **CRITICAL THINKING** Is it possible for an isosceles trapezoid to have two right angles? Explain.

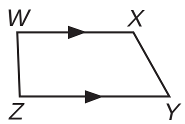
41. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are trapezoids used in architecture?

Include the following in your answer:
- the characteristics of a trapezoid, and
- other examples of trapezoids in architecture.

Standardized Test Practice

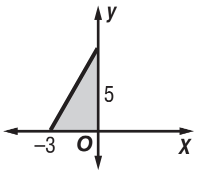
42. **SHORT RESPONSE** What type of quadrilateral is $WXYZ$? Justify your answer.



43. **ALGEBRA** In the figure, which point lies within the shaded region?

Ⓐ $(-2, 4)$ Ⓑ $(-1, 3)$

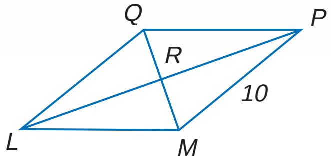
Ⓒ $(1, -3)$ Ⓓ $(2, -4)$



Maintain Your Skills

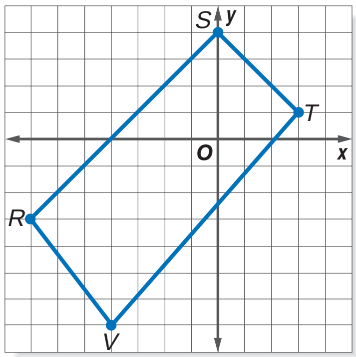
Mixed Review

ALGEBRA **In rhombus $LMPQ$, $m\angle QLM = 2x^2 - 10$, $m\angle QPM = 8x$, and $MP = 10$.** *(Lesson 8-5)*



44. Find $m\angle LPQ$.
45. Find QL.
46. Find $m\angle LQP$.
47. Find $m\angle LQM$.
48. Find the perimeter of $LMPQ$.

COORDINATE GEOMETRY **For Exercises 49–51, refer to quadrilateral $RSTV$.** *(Lesson 8-4)*



49. Find RS and TV.
50. Find the coordinates of the midpoints of $\overline{RT}$ and $\overline{SV}$.
51. Is $RSTV$ a rectangle? Explain.

Solve each proportion. *(Lesson 6-1)*

52. $\frac{16}{38} = \frac{24}{y}$ 53. $\frac{y}{6} = \frac{17}{30}$ 54. $\frac{5}{y+4} = \frac{20}{28}$ 55. $\frac{2y}{9} = \frac{52}{36}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Write an expression for the slope of the segment given the coordinates of the endpoints.** *(To review **slope**, see Lesson 3-3.)*

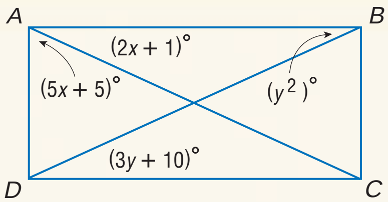
56. $(0, a), (-a, 2a)$
57. $(-a, b), (a, b)$
58. $(c, c), (c, d)$
59. $(a, -b), (2a, b)$
60. $(3a, 2b), (b, -a)$
61. $(b, c), (-b, -c)$

Practice Quiz 2

Lessons 8-4 through 8-6

Quadrilateral $ABCD$ is a rectangle. *(Lesson 8-4)*

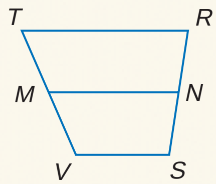
1. Find x.
2. Find y.



3. **COORDINATE GEOMETRY** Determine whether $MNPQ$ is a *rhombus*, a *rectangle*, or a *square* for $M(-5, -3)$, $N(-2, 3)$, $P(-2, -9)$, and $Q(1, -3)$. List all that apply. Explain. *(Lesson 8-5)*

For trapezoid $TRSV$, M and N are midpoints of the legs. *(Lesson 8-6)*

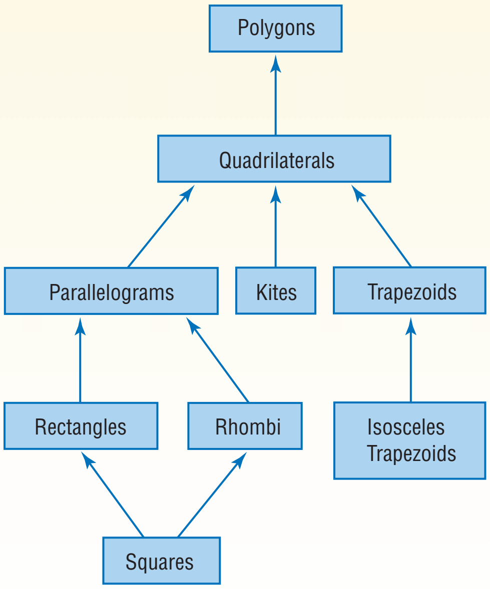
4. If $VS = 21$ and $TR = 44$, find MN.
5. If $TR = 32$ and $MN = 25$, find VS.



Reading Mathematics

Hierarchy of Polygons

A *hierarchy* is a ranking of classes or sets of things. Examples of some classes of polygons are rectangles, rhombi, trapezoids, parallelograms, squares, and quadrilaterals. These classes are arranged in the hierarchy below.



Use the following information to help read the hierarchy diagram.

- The class that is the broadest is listed first, followed by the other classes in order. For example, *polygons* is the broadest class in the hierarchy diagram above, and *squares* is a very specific class.
- Each class is contained within any class linked above it in the hierarchy. For example, *all* squares are also rhombi, rectangles, parallelograms, quadrilaterals, and polygons. However, an isosceles trapezoid is not a square or a kite.
- Some, but not all, elements of each class are contained within lower classes in the hierarchy. For example, some trapezoids are isosceles trapezoids, and some rectangles are squares.

Reading to Learn

Refer to the hierarchy diagram at the right. Write *true*, *false*, or *not enough information* for each statement.

1. All mogs are jums.

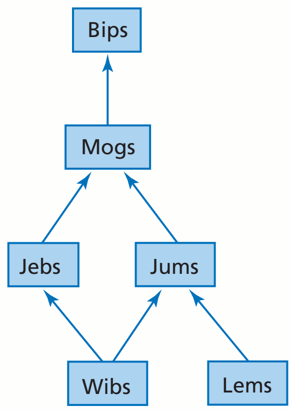
2. Some jebs are jums.

3. All lems are jums.

4. Some wibs are jums.

5. All mogs are bips.

6. Draw a hierarchy diagram to show these classes: equilateral triangles, polygons, isosceles triangles, triangles, and scalene triangles.



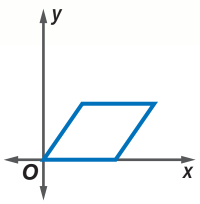
8-7 Coordinate Proof With Quadrilaterals

What You'll Learn

- Position and label quadrilaterals for use in coordinate proofs.
- Prove theorems using coordinate proofs.

How can you use a coordinate plane to prove theorems about quadrilaterals?

In Chapter 4, you learned that variable coordinates can be assigned to the vertices of triangles. Then the Distance and Midpoint Formulas and coordinate proofs were used to prove theorems. The same is true for quadrilaterals.



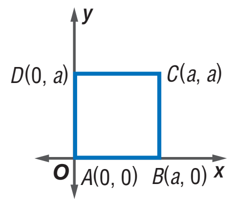
POSITION FIGURES The first step to using a coordinate proof is to place the figure on the coordinate plane. The placement of the figure can simplify the steps of the proof.

> **Study Tip**
>
> *Look Back*
> To review **placing a figure on a coordinate plane**, see Lesson 4-7.

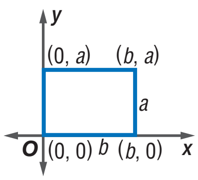
Example 1 Positioning a Square

Position and label a square with sides a units long on the coordinate plane.

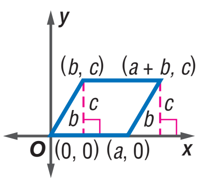
- Let A, B, C, and D be vertices of a square with sides a units long.
- Place the square with vertex A at the origin, $\overline{AB}$ along the positive x-axis, and $\overline{AD}$ along the y-axis. Label the vertices A, B, C, and D.
- The y-coordinate of B is 0 because the vertex is on the x-axis. Since the side length is a, the x-coordinate is a.
- D is on the y-axis so the x-coordinate is 0. The y-coordinate is $0 + a$ or a.
- The x-coordinate of C is also a. The y-coordinate is $0 + a$ or a because the side $\overline{BC}$ is a units long.



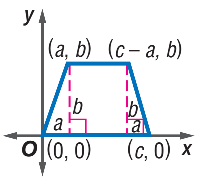
Some examples of quadrilaterals placed on the coordinate plane are given below. Notice how the figures have been placed so the coordinates of the vertices are as simple as possible.



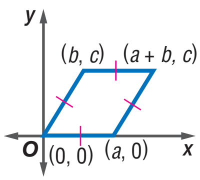
rectangle



parallelogram

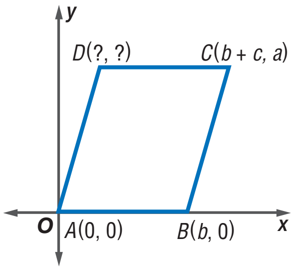


isosceles trapezoid



rhombus

Example 2 Find Missing Coordinates



Name the missing coordinates for the parallelogram.

Opposite sides of a parallelogram are congruent and parallel. So, the y-coordinate of D is a.

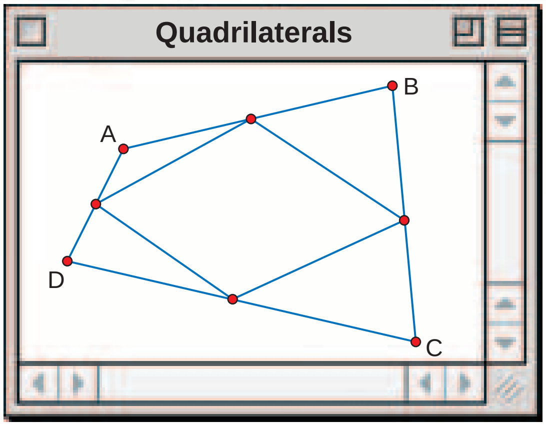
The length of $\overline{AB}$ is b, and the length of $\overline{DC}$ is b. So, the x-coordinate of D is $(b + c) - b$ or c.

The coordinates of D are (c, a).

PROVE THEOREMS Once a figure has been placed on the coordinate plane, we can prove theorems using the Slope, Midpoint, and Distance Formulas.

Geometry Software Investigation

Quadrilaterals



Model

- Use The Geometer's Sketchpad to draw a quadrilateral *ABCD* with no two sides parallel or congruent.
- Construct the midpoints of each side.
- Draw the quadrilateral formed by the midpoints of the segments.

Analyze

1. Measure each side of the quadrilateral determined by the midpoints of *ABCD*.
2. What type of quadrilateral is formed by the midpoints? Justify your answer.

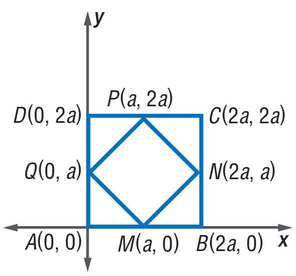
In this activity, you discover that the quadrilateral formed from the midpoints of any quadrilateral is a parallelogram. You will prove this in Exercise 22.

Study Tip

Problem Solving
To prove that a quadrilateral is a square, you can also show that all sides are congruent and that the diagonals bisect each other.

Example 3 Coordinate Proof

Place a square on a coordinate plane. Label the midpoints of the sides, *M, N, P,* and *Q*. Write a coordinate proof to prove that *MNPQ* is a square.



The first step is to position a square on the coordinate plane. Label the vertices to make computations as simple as possible.

Given: *ABCD* is a square.
M, N, P, and *Q* are midpoints.

Prove: *MNPQ* is a square.

Proof:

By the Midpoint Formula, the coordinates of M, N, P, and Q are as follows.

$$M\left(\frac{2a + 0}{2}, \frac{0 + 0}{2}\right) = (a, 0) \qquad N\left(\frac{2a + 2a}{2}, \frac{2a + 0}{2}\right) = (2a, a)$$

$$P\left(\frac{0 + 2a}{2}, \frac{2a + 2a}{2}\right) = (a, 2a) \qquad Q\left(\frac{0 + 0}{2}, \frac{0 + 2a}{2}\right) = (0, a)$$

Find the slopes of $\overline{QP}$, $\overline{MN}$, $\overline{QM}$, and $\overline{PN}$.

slope of $\overline{QP} = \frac{2a - a}{a - 0}$ or 1

slope of $\overline{MN} = \frac{a - 0}{2a - a}$ or 1

slope of $\overline{QM} = \frac{a - 0}{0 - a}$ or -1

slope of $\overline{PN} = \frac{2a - a}{a - 2a}$ or -1

Each pair of opposite sides is parallel, so they have the same slope. Consecutive sides form right angles because their slopes are negative reciprocals.

Use the Distance Formula to find the length of $\overline{QP}$ and $\overline{QM}$.

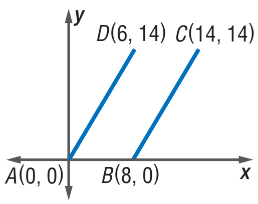
$$QP = \sqrt{(a - 0)^2 + (2a - a)^2} = \sqrt{a^2 + a^2} = \sqrt{2a^2} \text{ or } a\sqrt{2}$$

$$QM = \sqrt{(a - 0)^2 + (0 - a)^2} = \sqrt{a^2 + a^2} = \sqrt{2a^2} \text{ or } a\sqrt{2}$$

MNPQ is a square because each pair of opposite sides is parallel, and consecutive sides form right angles and are congruent.

Example 4 Properties of Quadrilaterals

PARKING **Write a coordinate proof to prove that the sides of the parking space are parallel.**



Given: $A(0, 0)$, $B(8, 0)$, $C(14, 14)$, $D(6, 14)$

Prove: $\overline{AD} \parallel \overline{BC}$

Proof:

slope of $\overline{AD} = \frac{14 - 0}{6 - 0}$ or $\frac{7}{3}$

slope of $\overline{BC} = \frac{14 - 0}{14 - 8}$ or $\frac{7}{3}$

Since $\overline{AD}$ and $\overline{BC}$ have the same slope, they are parallel.

Check for Understanding

Concept Check

1. **Explain** how to position a quadrilateral to simplify the steps of the proof.
2. **OPEN ENDED** Position and label a trapezoid with two vertices on the *y*-axis.

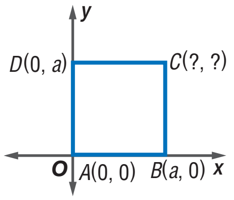
Guided Practice

Position and label the quadrilateral on the coordinate plane.

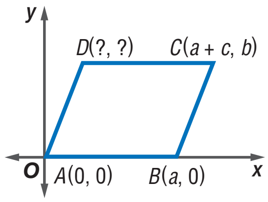
3. rectangle with length a units and height $a + b$ units

Name the missing coordinates for each quadrilateral.

4.



5.

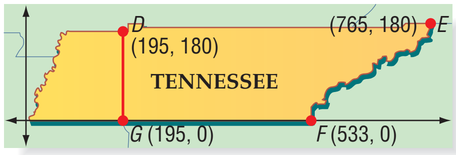


Write a coordinate proof for each statement.

6. The diagonals of a parallelogram bisect each other.
7. The diagonals of a square are perpendicular.

Application

8. **STATES** The state of Tennessee can be separated into two shapes that resemble quadrilaterals. Write a coordinate proof to prove that *DEFG* is a trapezoid. All measures are approximate and given in kilometers.



Practice and Apply

Homework Help	
For Exercises	See Examples
9–10	1
11–16, 23	2
17–22	3
24–26	4

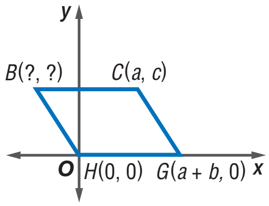
Extra Practice
See page 771.

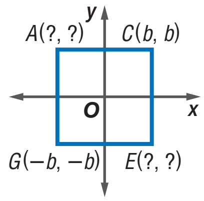
Position and label each quadrilateral on the coordinate plane.

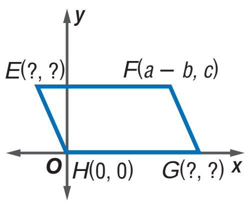
9. isosceles trapezoid with height c units, bases a units and $a + 2b$ units

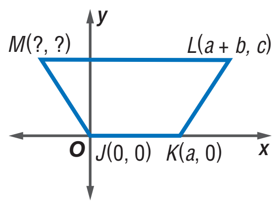
10. parallelogram with side length c units and height b units

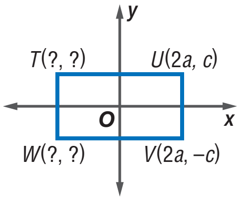
Name the missing coordinates for each quadrilateral.

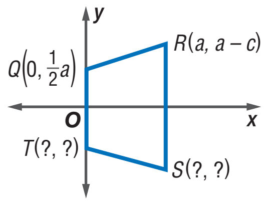
11.


12.


13.


14.


15.


16.


Position and label each figure on the coordinate plane. Then write a coordinate proof for each of the following.

17. The diagonals of a rectangle are congruent.

18. If the diagonals of a parallelogram are congruent, then it is a rectangle.

19. The diagonals of an isosceles trapezoid are congruent.

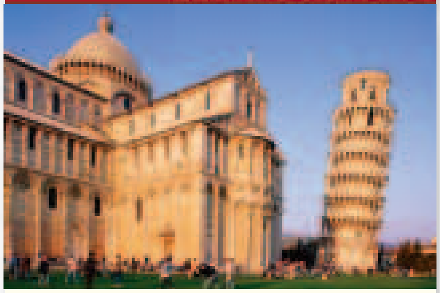
20. The median of a trapezoid is parallel to the bases.

21. The segments joining the midpoints of the sides of a rectangle form a rhombus.

22. The segments joining the midpoints of the sides of a quadrilateral form a parallelogram.

23. **CRITICAL THINKING** A has coordinates (0, 0), and B has coordinates (a, b). Find the coordinates of C and D so $ABCD$ is an isosceles trapezoid.

More About. . .



Architecture

The tower is also sinking. In 1838, the foundation was excavated to reveal the bases of the columns.

Source: www.torre.duomo.pisa.it

ARCHITECTURE **For Exercises 24–26, use the following information.**

The Leaning Tower of Pisa is approximately 60 meters tall, from base to belfry. The tower leans about 5.5° so the top level is 4.5 meters over the first level.

24. Position and label the tower on a coordinate plane.

25. Is it possible to write a coordinate proof to prove that the sides of the tower are parallel? Explain.

26. From the given information, what conclusion can be drawn?

27. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

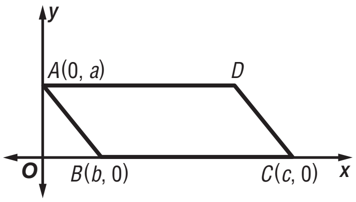
How is the coordinate plane used in proofs?

Include the following in your answer:

- guidelines for placing a figure on a coordinate grid, and
- an example of a theorem from this chapter that could be proved using the coordinate plane.

Standardized Test Practice

28. In the figure, *ABCD* is a parallelogram. What are the coordinates of point *D*?



Ⓐ $(a, c + b)$ Ⓑ $(c + b, a)$

Ⓒ $(b - c, a)$ Ⓓ $(c - b, a)$

29. ALGEBRA If $p = -5$, then $5 - p^2 - p =$ __?__.

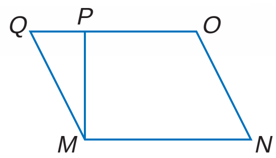
Ⓐ −15 Ⓑ −5

Ⓒ 10 Ⓓ 30

Maintain Your Skills

Mixed Review

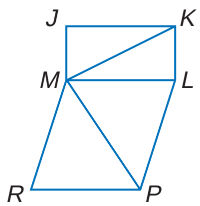
30. PROOF Write a two-column proof. *(Lesson 8-6)*



Given: *MNOP* is a trapezoid with bases $\overline{MN}$ and $\overline{OP}$. $\overline{MN} \cong \overline{QO}$

Prove: *MNOQ* is a parallelogram.

***JKLM* is a rectangle. *MLPR* is a rhombus. $\angle JMK \cong \angle RMP$, $m\angle JMK = 55$, and $m\angle MRP = 70$.** *(Lesson 8-5)*



31. Find $m\angle MPR$.

32. Find $m\angle KML$.

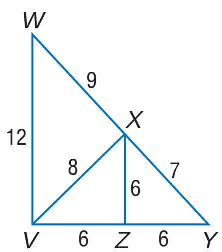
33. Find $m\angle KLP$.

Find the geometric mean between each pair of numbers. *(Lesson 7-1)*

34. 7 and 14

35. $2\sqrt{5}$ and $6\sqrt{5}$

Write an expression relating the given pair of angle measures. *(Lesson 5-5)*



36. $m\angle WVX, m\angle VXY$

37. $m\angle XVZ, m\angle VXZ$

38. $m\angle XYV, m\angle VXY$

39. $m\angle XZY, m\angle XZV$

www.geometryonline.com/self_check_quiz

Chapter 8 Study Guide and Review

Vocabulary and Concept Check

diagonal (p. 404)	median (p. 440)	rhombus (p. 431)
isosceles trapezoid (p. 439)	parallelogram (p. 411)	square (p. 432)
kite (p. 438)	rectangle (p. 424)	trapezoid (p. 439)

A complete list of postulates and theorems can be found on pages R1–R8.

Exercises **State whether each sentence is *true* or *false*. If false, replace the underlined term to make a true sentence.**

1. The diagonals of a rhombus are perpendicular.
2. All squares are rectangles.
3. If a parallelogram is a rhombus, then the diagonals are congruent.
4. Every parallelogram is a quadrilateral.
5. A(n) rhombus is a quadrilateral with exactly one pair of parallel sides.
6. Each diagonal of a rectangle bisects a pair of opposite angles.
7. If a quadrilateral is both a rhombus and a rectangle, then it is a square.
8. Both pairs of base angles in a(n) isosceles trapezoid are congruent.

Lesson-by-Lesson Review

8-1 Angles of Polygons

See pages 404–409.

Concept Summary

- If a convex polygon has n sides and the sum of the measures of its interior angles is S, then $S = 180(n - 2)$.
- The sum of the measures of the exterior angles of a convex polygon is 360.

Example **Find the measure of an interior angle of a regular decagon.**

$S = 180(n - 2)$ Interior Angle Sum Theorem

$= 180(10 - 2)$ $n = 10$

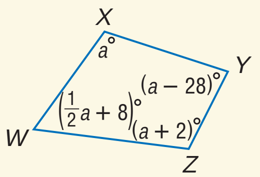
$= 180(8)$ or 1440 Simplify.

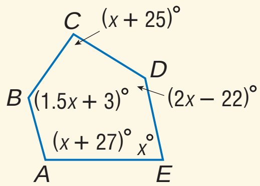
The measure of each interior angle is 1440 ÷ 10, or 144.

Exercises **Find the measure of each interior angle of a regular polygon given the number of sides.** *See Example 1 on page 405.*

9. 6
10. 15
11. 4
12. 20

ALGEBRA **Find the measure of each interior angle.** *See Example 3 on page 405.*

13.


14.


www.geometryonline.com/vocabulary_review

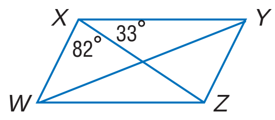
8-2 Parallelograms

See pages 411–416.

Concept Summary

- In a parallelogram, opposite sides are parallel and congruent, opposite angles are congruent, and consecutive angles are supplementary.
- The diagonals of a parallelogram bisect each other.

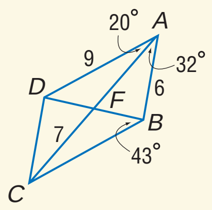
Example **WXYZ is a parallelogram. Find $m\angle YZW$ and $m\angle XWZ$.**



$m\angle YZW = m\angle WXY$	Opp. ∡ of ▱ are ≅.
$m\angle YZW = 82 + 33$ or 115	$m\angle WXY = m\angle WXZ + m\angle YXZ$
$m\angle XWZ + m\angle WXY = 180$	Cons. ∡ in ▱ are suppl.
$m\angle XWZ + (82 + 33) = 180$	$m\angle WXY = m\angle WXZ + m\angle YXZ$
$m\angle XWZ + 115 = 180$	Simplify.
$m\angle XWZ = 65$	Subtract 115 from each side.

Exercises **Use ▱ABCD to find each measure.**

See Example 2 on page 413.



15. $m\angle BCD$ **16.** AF
17. $m\angle BDC$ **18.** BC
19. CD **20.** $m\angle ADC$

8-3 Tests for Parallelograms

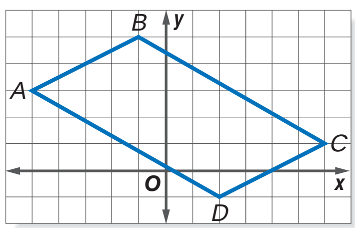
See pages 417–423.

Concept Summary

A quadrilateral is a parallelogram if any one of the following is true.

- Both pairs of opposite sides are parallel and congruent.
- Both pairs of opposite angles are congruent.
- Diagonals bisect each other.
- A pair of opposite sides is both parallel and congruent.

Example **COORDINATE GEOMETRY** **Determine whether the figure with vertices $A(-5, 3)$, $B(-1, 5)$, $C(6, 1)$, and $D(2, -1)$ is a parallelogram. Use the Distance and Slope Formulas.**



$$AB = \sqrt{[-5-(-1)]^2 + (3-5)^2}$$
$$= \sqrt{(-4)^2 + (-2)^2} \text{ or } \sqrt{20}$$
$$CD = \sqrt{(6-2)^2 + [1-(-1)]^2}$$
$$= \sqrt{4^2 + 2^2} \text{ or } \sqrt{20}$$

Since $AB = CD$, $\overline{AB} \cong \overline{CD}$.

slope of $\overline{AB} = \frac{5-3}{-1-(-5)}$ or $\frac{1}{2}$ slope of $\overline{CD} = \frac{-1-1}{2-6}$ or $\frac{1}{2}$

$\overline{AB}$ and $\overline{CD}$ have the same slope, so they are parallel. Since one pair of opposite sides is congruent and parallel, $ABCD$ is a parallelogram.

Exercises **Determine whether the figure with the given vertices is a parallelogram. Use the method indicated.** *See Example 5 on page 420.*

21. $A(-2, 5)$, $B(4, 4)$, $C(6, -3)$, $D(-1, -2)$; Distance Formula

22. $H(0, 4)$, $J(-4, 6)$, $K(5, 6)$, $L(9, 4)$; Midpoint Formula

23. $S(-2, -1)$, $T(2, 5)$, $V(-10, 13)$, $W(-14, 7)$; Slope Formula

8-4 Rectangles

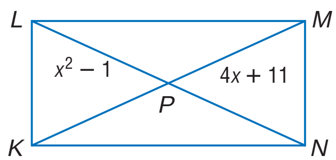
See pages 424–430.

Concept Summary

- A rectangle is a quadrilateral with four right angles and congruent diagonals.
- If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Example

Quadrilateral *KLMN* is a rectangle. If $PL = x^2 - 1$ and $PM = 4x + 11$, find x.



The diagonals of a rectangle are congruent and bisect each other, so $\overline{PL} \cong \overline{PM}$.

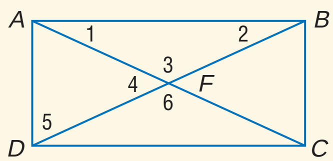
$\overline{PL} \cong \overline{PM}$	Diag. are ≅ and bisect each other.
$PL = PM$	Def. of ≅ angles
$x^2 - 1 = 4x + 11$	Substitution
$x^2 - 1 - 4x = 11$	Subtract 4*x* from each side.
$x^2 - 4x - 12 = 0$	Subtract 11 from each side.
$(x + 2)(x - 6) = 0$	Factor.

$x + 2 = 0$ $\qquad$ $x - 6 = 0$

$x = -2$ $\qquad$ $x = 6$

The value of x is -2 or 6.

Exercises ***ABCD* is a rectangle.**
See Examples 1 and 2 on pages 425 and 426.



24. If $AC = 26$ and $AF = 2x + 7$, find AF.

25. If $m\angle 1 = 52$ and $m\angle 2 = 16x - 12$, find $m\angle 2$.

26. If $CF = 4x + 1$ and $DF = x + 13$, find x.

27. If $m\angle 2 = 70 - 4x$ and $m\angle 5 = 18x - 8$, find $m\angle 5$.

COORDINATE GEOMETRY **Determine whether *RSTV* is a rectangle given each set of vertices. Justify your answer.** *See Example 4 on pages 426 and 427.*

28. $R(-3, -5)$, $S(0, -5)$, $T(0, 4)$, $V(3, 4)$

29. $R(0, 0)$, $S(6, 3)$, $T(-2, 4)$, $V(4, 7)$

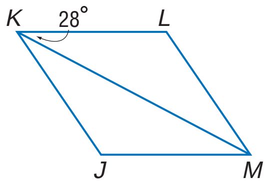
8-5 Rhombi and Squares

See pages 431–437.

Concept Summary

- A rhombus is a quadrilateral with each side congruent, diagonals that are perpendicular, and each diagonal bisecting a pair of opposite angles.
- A quadrilateral that is both a rhombus and a rectangle is a square.

Example **Use rhombus $JKLM$ to find $m\angle JMK$ and $m\angle KJM$.**



The opposite sides of a rhombus are parallel, so $\overline{KL} \parallel \overline{JM}$. $\angle JMK \cong \angle LKM$ because alternate interior angles are congruent.

$m\angle JMK = m\angle LKM$ Definition of congruence

$= 28$ Substitution

The diagonals of a rhombus bisect the angles, so $\angle JKM \cong \angle LKM$.

$m\angle KJM + m\angle JKL = 180$ Cons. ∠s in ▱ are suppl.

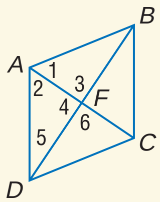
$m\angle KJM + (m\angle JKM + m\angle LKM) = 180$ $m\angle JKL = m\angle JKM + m\angle LKM$

$m\angle KJM + (28 + 28) = 180$ Substitution

$m\angle KJM + 56 = 180$ Add.

$m\angle KJM = 124$ Subtract 56 from each side.

Exercises **Use rhombus $ABCD$ with $m\angle 1 = 2x + 20$, $m\angle 2 = 5x - 4$, $AC = 15$, and $m\angle 3 = y^2 + 26$.** *See Example 2 on page 432.*



30. Find x.

31. Find AF.

32. Find y.

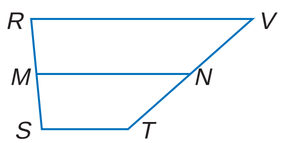
8-6 Trapezoids

See pages 439–445.

Concept Summary

- In an isosceles trapezoid, both pairs of base angles are congruent and the diagonals are congruent.
- The median of a trapezoid is parallel to the bases, and its measure is one-half the sum of the measures of the bases.

Example **$RSTV$ is a trapezoid with bases $\overline{RV}$ and $\overline{ST}$ and median $\overline{MN}$. Find x if $MN = 60$, $ST = 4x - 1$, and $RV = 6x + 11$.**



$MN = \frac{1}{2}(ST + RV)$

$60 = \frac{1}{2}[(4x - 1) + (6x + 11)]$ Substitution

$120 = 4x - 1 + 6x + 11$ Multiply each side by 2.

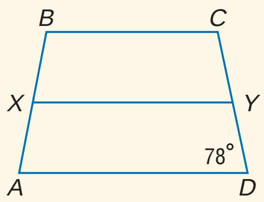
$120 = 10x + 10$ Simplify.

$110 = 10x$ Subtract 10 from each side.

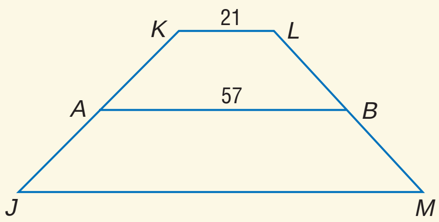
$11 = x$ Divide each side by 10.

Exercises **Find the missing value for the given trapezoid.**
See Example 4 on page 441.

33. For isosceles trapezoid $ABCD$, X and Y are midpoints of the legs. Find $m\angle XBC$ if $m\angle ADY = 78$.



34. For trapezoid $JKLM$, A and B are midpoints of the legs. If $AB = 57$ and $KL = 21$, find JM.



8-7 Coordinate Proof with Quadrilaterals

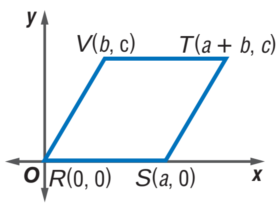
See pages 447–451.

Concept Summary

- Position a quadrilateral so that a vertex is at the origin and at least one side lies along an axis.

Example **Position and label rhombus *RSTV* on the coordinate plane. Then write a coordinate proof to prove that each pair of opposite sides is parallel.**

First, draw rhombus $RSTV$ on the coordinate plane. Label the coordinates of the vertices.



Given: $RSTV$ is a rhombus.

Prove: $\overline{RV} \parallel \overline{ST}$, $\overline{RS} \parallel \overline{VT}$

Proof:

slope of $\overline{RV} = \frac{c - 0}{b - 0}$ or $\frac{c}{b}$ slope of $\overline{ST} = \frac{c - 0}{(a + b) - a}$ or $\frac{c}{b}$

slope of $\overline{RS} = \frac{0 - 0}{a - 0}$ or 0 slope of $\overline{VT} = \frac{c - c}{(a + b) - b}$ or 0

$\overline{RV}$ and $\overline{ST}$ have the same slope. So $\overline{RV} \parallel \overline{ST}$. $\overline{RS}$ and $\overline{VT}$ have the same slope, and $\overline{RS} \parallel \overline{VT}$.

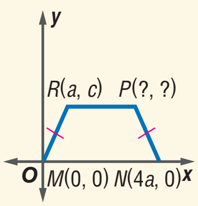
Exercises **Position and label each figure on the coordinate plane. Then write a coordinate proof for each of the following.** *See Example 3 on pages 448 and 449.*

35. The diagonals of a square are perpendicular.

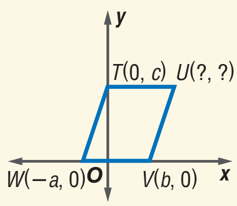
36. A diagonal separates a parallelogram into two congruent triangles.

Name the missing coordinates for each quadrilateral. *See Example 2 on page 448.*

37.



38.



Chapter 8 Practice Test

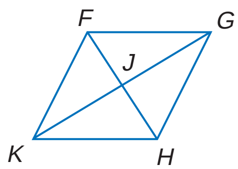
Vocabulary and Concepts

Determine whether each conditional is *true* or *false*. If false, draw a counterexample.

1. If a quadrilateral has four right angles, then it is a rectangle.
2. If a quadrilateral has all four sides congruent, then it is a square.
3. If the diagonals of a quadrilateral are perpendicular, then it is a rhombus.

Skills and Applications

Complete each statement about ▱*FGHK*. Justify your answer.

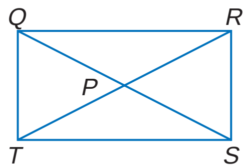


4. $\overline{HK} \cong$ __?__.
5. $\angle FKJ \cong$ __?__.
6. $\angle FKH \cong$ __?__.
7. $\overline{GH} \parallel$ __?__.

Determine whether the figure with the given vertices is a parallelogram. Justify your answer.

8. $A(4, 3), B(6, 0), C(4, -8), D(2, -5)$
9. $S(-2, 6), T(2, 11), V(3, 8), W(-1, 3)$
10. $F(7, -3), G(4, -2), H(6, 4), J(12, 2)$
11. $W(-4, 2), X(-3, 6), Y(2, 7), Z(1, 3)$

ALGEBRA ***QRST*** **is a rectangle.**



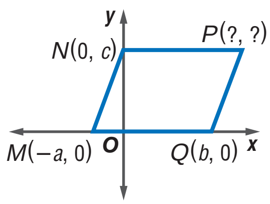
12. If $QP = 3x + 11$ and $PS = 4x + 8$, find QS.
13. If $m\angle QTR = 2x^2 - 7$ and $m\angle SRT = x^2 + 18$, find $m\angle QTR$.

COORDINATE GEOMETRY **Determine whether ▱*ABCD* is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain your reasoning.**

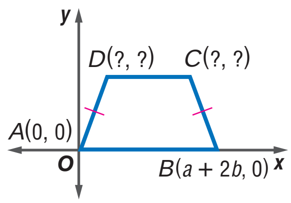
14. $A(12, 0), B(6, -6), C(0, 0), D(6, 6)$
15. $A(-2, 4), B(5, 6), C(12, 4), D(5, 2)$

Name the missing coordinates for each quadrilateral.

16.

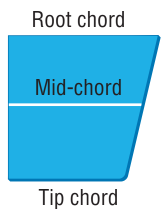


17.



18. Position and label a trapezoid on the coordinate plane. Write a coordinate proof to prove that the median is parallel to each base.

19. **SAILING** Many large sailboats have a *keel* to keep the boat stable in high winds. A keel is shaped like a trapezoid with its top and bottom parallel. If the root chord is 9.8 feet and the tip chord is 7.4 feet, find the length of the mid-chord.



20. **STANDARDIZED TEST PRACTICE** The measure of an interior angle of a regular polygon is 108. Find the number of sides.

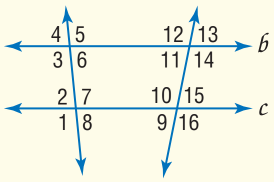
(A) 8 (B) 6 (C) 5 (D) 3

Chapter 8 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. A trucking company wants to purchase a ramp to use when loading heavy objects onto a truck. The closest that the truck can get to the loading area is 5 meters. The height from the ground to the bed of the truck is 3 meters. To the nearest meter, what should the length of the ramp be? (Lesson 1-3)



Ⓐ 4 m Ⓑ 5 m

Ⓒ 6 m Ⓓ 7 m

2. Which of the following is the contrapositive of the statement below? (Lesson 2-3)

If an astronaut is in orbit, then he or she is weightless.

Ⓐ If an astronaut is weightless, then he or she is in orbit.

Ⓑ If an astronaut is not in orbit, then he or she is not weightless.

Ⓒ If an astronaut is on Earth, then he or she is weightless.

Ⓓ If an astronaut is not weightless, then he or she is not in orbit.

3. Rectangle *QRST* measures 7 centimeters long and 4 centimeters wide. Which of the following could be the dimensions of a rectangle similar to rectangle *QRST*? (Lesson 6-2)

Ⓐ 28 cm by 14 cm

Ⓑ 21 cm by 12 cm

Ⓒ 14 cm by 4 cm

Ⓓ 7 cm by 8 cm

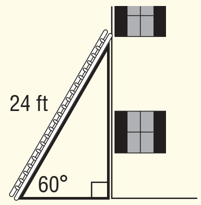
4. A 24 foot ladder, leaning against a house, forms a 60° angle with the ground. How far up the side of the house does the ladder reach? (Lesson 7-3)

Ⓐ 12 ft

Ⓑ $12\sqrt{2}$ ft

Ⓒ $12\sqrt{3}$ ft

Ⓓ 20 ft



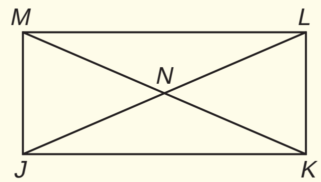
5. In rectangle *JKLM* shown below, $\overline{JL}$ and $\overline{MK}$ are diagonals. If $JL = 2x + 5$ and $KM = 4x - 11$, what is x? (Lesson 8-4)

Ⓐ 10

Ⓑ 8

Ⓒ 6

Ⓓ 5

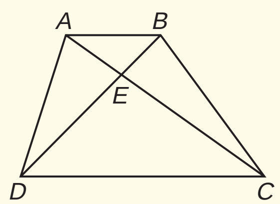


6. Joaquin bought a set of stencils for his younger sister. One of the stencils is a quadrilateral with perpendicular diagonals that bisect each other, but are **not** congruent. What kind of quadrilateral is this piece? (Lesson 8-5)

Ⓐ square Ⓑ rectangle

Ⓒ rhombus Ⓓ trapezoid

7. In the diagram below, *ABCD* is a trapezoid with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at point *E*.



Which statement is true? (Lesson 8-6)

Ⓐ $\overline{AB}$ is parallel to $\overline{CD}$.

Ⓑ $\angle ADC$ is congruent to $\angle BCD$.

Ⓒ $\overline{CE}$ is congruent to $\overline{DE}$.

Ⓓ $\overline{AC}$ and $\overline{BD}$ bisect each other.

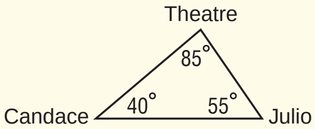
Aligned and verified by The Princeton Review

Part 2 Short Response/Grid In

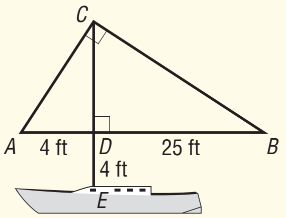
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. At what point does the graph of $y = -4x + 5$ cross the x-axis on a coordinate plane? (Prerequisite Skill)

9. Candace and Julio are planning to see a movie together. They decide to meet at the house that is closer to the theater. From the locations shown on the diagram, whose house is closer to the theater? (Lesson 5-3)

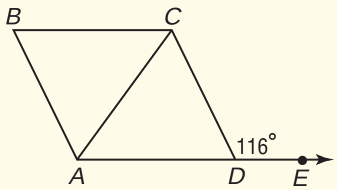


10. In the diagram, $\overline{CE}$ is the mast of a sailboat with sail $\triangle ABC$.



Marcia wants to calculate the length, in feet, of the mast. Write an equation in which the geometric mean is represented by x. (Lesson 7-1)

11. $\overline{AC}$ is a diagonal of rhombus $ABCD$. If $m\angle CDE$ is 116, what is $m\angle ACD$? (Lesson 8-4)



Test-Taking Tip

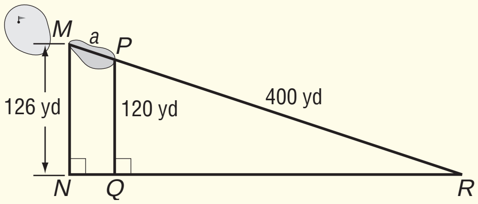
The Princeton Review

Question 10 Read the question carefully to check that you answered the question that was asked. In Question 10, you are asked to write an equation, not to find the length of the mast.

Part 3 Open Ended

Record your answers on a sheet of paper. Show your work.

12. On the tenth hole of a golf course, a sand trap is located right before the green at point M. Matt is standing 126 yards away from the green at point N. Quintashia is standing 120 yards away from the beginning of the sand trap at point Q.



a. Explain why $\triangle MNR$ is similar to $\triangle PQR$. (Lesson 6-3)

b. Write and solve a proportion to find the distance across the sand trap, a. (Lesson 6-3)

13. Quadrilateral $ABCD$ has vertices with coordinates: $A(0, 0)$, $B(a, 0)$, $C(a + b, c)$, and $D(b, c)$.

a. Position and label $ABCD$ on the coordinate plane. Prove that $ABCD$ is a parallelogram. (Lesson 8-2 and 8-7)

b. If $a^2 = b^2 + c^2$, what can you determine about the slopes of the diagonals $\overline{AC}$ and $\overline{BD}$? (Lesson 8-7)

c. What kind of parallelogram is $ABCD$? (Lesson 8-7)

www.geometryonline.com/standardized_test